

Math115 Midterm Revision Questions:

1. Diagonalise this matrix A and hence find A^5 .

$$\begin{bmatrix} 5 & 18 & -15 \\ 24 & 71 & -60 \\ 30 & 90 & -76 \end{bmatrix}$$

2. Find the determinant of this matrix:

$$\begin{bmatrix} -1 & 4 & u \\ v & -3 & 1 \\ 1 & -2 & 0 \end{bmatrix}$$

When is it singular?

3. If we have this matrix

$$L := \begin{bmatrix} -1 & 2 & -1 & -6 & -1 \\ 1 & 1 & 4 & -12 & 3 \\ 2 & -4 & 2 & 12 & 2 \end{bmatrix}$$

what are the solutions to $Lx = \begin{bmatrix} -23 \\ -32 \\ 46 \end{bmatrix}$ and $L^T y = \begin{bmatrix} 13 \\ -11 \\ 28 \\ -12 \\ 23 \end{bmatrix}$?

4. Find the eigenvalues and eigenvectors of $F := \begin{bmatrix} -4 & 2 \\ -3 & 1 \end{bmatrix}$. Evaluate F^2 and find its eigenvectors and eigenvalues too.
5. By using $Av = \lambda v$, substitute twice for Av in A^2v and hence prove that v is also an eigenvector for A^2 and its eigenvalue is λ^2 .
6. Find two example of non-diagonal matrices which are self-inverse which are 2×2 and 3×3 , and then give a pattern for one which is $n \times n$. Is it true that the product of any two self inverse matrices of the same size is also self inverse?

7. Use the adjoint formula to find the inverse of $\begin{bmatrix} a & b & 0 & 0 \\ c & d & 0 & 0 \\ 0 & 0 & e & f \\ 0 & 0 & g & h \end{bmatrix}$. What is the determinant?