University College of Cape Breton MATH115

Matrix Algebra

April 2005
Time : 3 hours

Student Name:
Registration Number:

Full marks will be obtained by perfect answers to FIVE questions, please make sure to give all reasoning and working for all questions answered.

Q1. (a) Find the inverse of this matrix using row operations. [10]

$$A := \begin{pmatrix} 0 & -1 & -1 & -1 \\ 0 & 1 & -1 & 1 \\ -1 & -1 & 1 & 1 \\ 1 & 0 & 1 & -1 \end{pmatrix}$$

(b) Deduce the null space and image space of A. [2]

Q2. (a) Determine the eigenvalues and eigenvectors of this matrix [10]

$$C := \frac{1}{40}\begin{pmatrix} 38 & -12 & 12 \\ -117 & -22 & -18 \\ -234 & 36 & -76 \end{pmatrix}$$

(b) Calculate $C\begin{pmatrix} 2 \\ 6 \\ -7 \end{pmatrix}$ and explain why we get this relation. [2]

Q3. (a) What is the intersection of these two planes? Check your answer. [7]

$$P_1 : \begin{pmatrix} x \\ y \\ z \end{pmatrix} \circ \begin{pmatrix} 3 \\ 4 \\ 2 \end{pmatrix} = -5 \ , \quad P_2 : \begin{pmatrix} x \\ y \\ z \end{pmatrix} \circ \begin{pmatrix} -5 \\ -2 \\ 6 \end{pmatrix} = 3$$

(b) Where does the line $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ 0 \\ -1 \end{pmatrix} + k\begin{pmatrix} 2 \\ -7 \\ 4 \end{pmatrix}$ pass through the planes? [3]

(c) Give the distance from the origin to the intersection point in the previous part. [2]

Q4. (a) What is the quadratic equation which best fits this data? [8]

x_i	-2	-1	1	2
y_i	$\frac{1}{2}$	$\frac{-7}{2}$	$\frac{-1}{2}$	$\frac{3}{2}$

(b) How big is the difference between the quadratic and the given y_i? [2]

(c) Could there be a cubic curve which passes through all four points? [2]

Q5. (a) Explain why $Y := 2A - A^T$ is defined only for square matrices A and Y is symmmetric if and only if A is symmetric. [4]

(b) (i) Solve this equation in general: $(X(B^T - 3C))^T = 2B$. [2]

(ii) Find X and verify your answer is correct using these values: [3]

$$B := \begin{pmatrix} 1 & 3 \\ -1 & 2 \end{pmatrix}, \quad C := \begin{pmatrix} 2 & -1 \\ -4 & 2 \end{pmatrix}$$

(iii) Explain three different ways that the equation can have no solution. [3]

Q6. (a) Give $\underline{w} := \begin{pmatrix} -2 \\ 5 \\ 5 \\ 4 \end{pmatrix}$ in terms of $\underline{e}_1 := \begin{pmatrix} 2 \\ 5 \\ 1 \\ 2 \end{pmatrix}$ and $\underline{v}_2 := \begin{pmatrix} 4 \\ 5 \\ -1 \\ 1 \end{pmatrix}$ but show that $\underline{v}_3 := \begin{pmatrix} 2 \\ 3 \\ 0 \\ -1 \end{pmatrix}$ is not in the space spanned by $\underline{e}_1$ and $\underline{v}_2$. [6]

(b) Use Gram-Schmidt to find $\underline{e}_2$ from $\underline{v}_2$ and then find an $\underline{e}_3$ from $\underline{v}_3$ orthogonal to both $\underline{e}_1$ and $\underline{e}_2$ and check your answers. [6]

Q7. (a) What is the determinant of this matrix? $J := \begin{pmatrix} 2 & x & 4 \\ 2 & 2 & 1 \\ y & 1 & -1 \end{pmatrix}$ [4]

(b) Under what conditions for y will J be singular? Which x and y values will never give singular matrices? [4]

(c) Determine the inverse of J in the general case by the determinant method. [4]

Q8. (a) Check all three subspace axioms for this set: $\left\{\begin{pmatrix} x \\ y \end{pmatrix}; y = 2x - 6 \text{ or } y = \frac{x}{2}\right\}$. [4]

(b) Find the dot product form of the plane containing the points $u := \begin{pmatrix} 2 \\ 3 \\ 2 \end{pmatrix}$, $v := \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$ and $w := \begin{pmatrix} -1 \\ -2 \\ 1 \end{pmatrix}$. Find one reason why it is not a subspace of $\mathbb{R}^3$. [5]

(c) What is the dimension of the subspace with equation $w - 3x + 2y - 7z = 0$? [3]