University College of Cape Breton

Math115

MATRIX ALGEBRA

February 2005

Time : 1.5 hours

Answer any THREE of these questions, giving all of your working and reasoning.

Q1. (a) Find the determinant of this matrix [4]

$$C := \begin{pmatrix} 1 & -4 & x \\ 1 & -2 & -1 \\ x-3 & 3 & 4 \end{pmatrix}$$

(b) For which values of x is C singular? Calculate the rank of C for one of these values of x. Can the rank of C be 1 for any value of x? [6]

Q2. (a) Verify that $\begin{pmatrix} 5 \\ -3 \end{pmatrix}$ is an eigenvector of both of these matrices, but that the two eigenvalues are different. [3]

$$A := \begin{pmatrix} 13 & 10 \\ -3 & 2 \end{pmatrix} \quad , \quad B := \begin{pmatrix} 13 & 15 \\ -9 & -11 \end{pmatrix}$$

(b) Find one of the other eigenvectors and show it is not an eigenvector of the matrix it doesn't come from. [7]

Q3. (a) Solve this system of equations by using row operations or LU factorisation. [8]

$$\begin{aligned} w - 2x - y - z &= 1 \qquad & w - x - y &= 2 \\ w + x + 2y + 2z &= 1 \qquad & w + z &= 2 \end{aligned}$$

(b) Which part of your final answer is the solution to the corresponding set of homogeneous equations? Verify your answer. [2]

Q4. (a) Using the 2×2 formula, find $\det(EF)$ and check the expression is identical to $\det(E) \times \det(F)$. [5]

$$E := \begin{pmatrix} e_{1,1} & e_{1,2} \\ e_{2,1} & e_{2,2} \end{pmatrix} \ , \quad F := \begin{pmatrix} f_{1,1} & f_{1,2} \\ f_{2,1} & f_{2,2} \end{pmatrix}$$

(b) Use (a) to deduce that $\det(EF) = \det(FE)$. [1]

(c) Explain why we can deduce that $\det(E^{-1}) = \frac{1}{\det(E)}$ from the above work and check your answer using the general formula for the inverse of a 2×2 matrix. [4]

END OF QUESTION PAPER