

2005 Math115 Midterm Review Questions:

1. Diagonalise this matrix A and hence find A^5 .

$$\begin{pmatrix} 5 & 18 & -15 \\ 24 & 71 & -60 \\ 30 & 90 & -76 \end{pmatrix}$$

2. Find the determinant of this matrix:

$$\begin{pmatrix} -1 & 4 & u \\ v & -3 & 1 \\ 1 & -2 & 0 \end{pmatrix}$$

For which values of u and v is it singular?

3. If we have this matrix

$$B := \begin{pmatrix} -1 & 2 & -1 & -6 & -1 \\ 1 & 1 & 4 & -12 & 3 \\ 2 & -4 & 2 & 12 & 2 \end{pmatrix}$$

what are the solutions to $B^T X = \begin{pmatrix} 13 \\ -11 \\ 28 \\ -12 \\ 23 \end{pmatrix}$?

4. (a) Find the eigenvalues and eigenvectors of $F := \begin{pmatrix} -4 & 2 \\ -3 & 1 \end{pmatrix}$. Evaluate F^2 and find its eigenvectors and eigenvalues too.
- (b) By using $Av = \lambda v$, substitute twice for Av in A^2v and hence prove that v is also an eigenvector for A^2 and its eigenvalue is λ^2 .

5. Use the adjoint formula to find the inverse of $\begin{pmatrix} a & b & 0 & 0 \\ c & d & 0 & 0 \\ 0 & 0 & e & f \\ 0 & 0 & g & h \end{pmatrix}$. What is the determinant?

6. What is the LU-factorisation of G ? Solve $LV = B = \begin{pmatrix} 1 \\ -1 \\ -3 \\ 0 \end{pmatrix}$ and then $UW = V$ to find all solutions to the homogeneous equations $GW = B$. Check your answers are actually solutions. What is the rank of G ?

$$\begin{pmatrix} 2 & 2 & 1 & -2 & -2 & 0 \\ -1 & 1 & -1 & 1 & 2 & -2 \\ -1 & 0 & 1 & 0 & 2 & -1 \\ 1 & 3 & 0 & -1 & 0 & -2 \end{pmatrix}$$