Math 115 Test 4: Recurrences and Orthogonality

March 24, 2005

Each question is weighted as shown in square brackets, use the appropriate amount of time and space to answer all parts. Give *all* working and reasoning for your answers to achieve full marks.

1. (a) Given these two equations, find the values of f_1 and g_1 if $f_0 := 4$ and $g_0 := 11$. [2]

$$f_{i+1} := 3(20g_i - 17f_i) \ , \qquad 7g_{i+1} := 353g_i - 300f_i$$

   (b) Form a matrix relation between consecutive terms of the f and g sequences and extend that to relate the general f and g vector to f_0 and g_0. [2]

   (c) Verify that the eigenvectors of this matrix are $\begin{pmatrix} 6 \\ 5 \end{pmatrix}$ and $\begin{pmatrix} 7 \\ 6 \end{pmatrix}$ [1]

   (d) Use the diagonalisation of the matrix to find the value of the n^{th} f and g vector and check your formula against your calculated values for f_1 and g_1. [5]

   (e) What ratio does $\frac{f_n}{g_n}$ approximate and how is it related to the eigenvalues? [2]

2. (a) Given this plane, find a vector normal to it using row operations on a matrix and hence give its equation in dot product form. [4]

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -3 \\ 7 \\ 1 \end{pmatrix} + k \begin{pmatrix} -5 \\ 27 \\ 1 \end{pmatrix} + l \begin{pmatrix} 16 \\ 1 \\ -7 \end{pmatrix}$$

   (b) Verify that both the origin and $e_1 := \begin{pmatrix} -5 \\ 4 \\ 2 \end{pmatrix}$ actually lie in this plane. [1]

   (c) Find e_2 which is also in the plane but which is orthogonal to e_1. [3]

   (d) Check that the Gram Schmidt procedure with e_1, e_2 and $v_3 := \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ gives the normal from the beginning of the question. [4]