

# Math 115 Test 6: Vector Spaces and Approximations

March 29, 2005

Each question is weighted as shown in square brackets, use the appropriate amount of time and space to answer all parts. Give *all* working and reasoning for each question to achieve full marks.

The answers must be entirely your own work, and a statement to this effect should preface your answers. Plagiarism will be detected and the appropriate academic penalties enforced.

1. (a) Prove that  $v_1 := \begin{pmatrix} 2 \\ -1 \\ 3 \\ -2 \end{pmatrix}$ ,  $v_2 := \begin{pmatrix} 1 \\ -3 \\ 2 \\ -1 \end{pmatrix}$  and  $v_3 := \begin{pmatrix} -1 \\ 2 \\ -1 \\ 1 \end{pmatrix}$  are linearly independent, using row operations. [6]

- (b) Re-use these operations to show that  $v_4 := \begin{pmatrix} 4 \\ -5 \\ 6 \\ -4 \end{pmatrix}$  is in the space spanned by  $v_1, v_2$  and  $v_3$  and verify that your values so obtained for  $k_1, k_2$  and  $k_3$  relate the vectors. [4]

- (c) Use determinants to prove that  $v_5 := \begin{pmatrix} 1 \\ 0 \\ -1 \\ 0 \end{pmatrix}$  is not in  $\text{span}(v_1, v_2, v_3)$  but  $v_6 := \begin{pmatrix} 0 \\ 1 \\ -1 \\ 0 \end{pmatrix}$  is. [6]

- (d) What is the dimension of the space spanned by  $v_1, v_2, v_3, v_4, v_5$  and  $v_6$ ? [2]

2. Calculate the Null Space and Image Space of this matrix [10]

$$B := \begin{pmatrix} -3 & -4 & -2 \\ 10 & 7 & 9 \\ -4 & 1 & -5 \end{pmatrix}$$

3. (a) Calculate the straight line which passes nearest to these points: [6]

$$\begin{array}{c|cccc} x_i & 0 & 2 & -3 & -1 \\ \hline y_i & -1 & 3 & 5 & 2 \end{array}$$

- (b) Find all quadratics which pass exactly through the points  $(-1,4)$  and  $(3,0)$ . [6]