

Math115 Test 5

March 31, 2007

Answer all questions and give complete reasons and checks for your answers. The parts of the questions are weighted as shown. The questions can be answered in any order, please start a fresh sheet of paper for each question. You are reminded that plagiarism is a serious offense and when caught you will suffer the penalties specified by the University.

Taking your registration number as a set of 8 digits, remove all the zeros and let the last four non-zero digits be the letters a , b , c and d in order. For example, if my registration number was 40062013 my values would be $a := 6$, $b := 2$, $c := 1$ and $d := 3$.

1. Find the shortest distance from the point $\begin{pmatrix} a \\ b \\ c \end{pmatrix}$ to the line $\begin{pmatrix} -1 \\ 3 \\ 2 \end{pmatrix} + k \begin{pmatrix} d \\ -2 \\ 1 \end{pmatrix}$. [4]

2. Determine at which point the lines $\begin{pmatrix} 1 \\ -3 \end{pmatrix} + l \begin{pmatrix} a \\ d \end{pmatrix}$ and $\begin{pmatrix} 2 \\ -1 \end{pmatrix} + m \begin{pmatrix} b \\ c \end{pmatrix}$ intersect. [4]

3. Find a vector equation for the plane $\begin{pmatrix} d \\ c \\ b \end{pmatrix} \circ \begin{pmatrix} x \\ y \\ z \end{pmatrix} = a$ and determine whether or not the point $\begin{pmatrix} -3 \\ 4 \\ -2 \end{pmatrix}$ lies on the plane. [3]

4. (a) Use the Gram-Schmidt process with $v_1 := \begin{pmatrix} 3 \\ 0 \\ -2 \end{pmatrix}$ and $v_2 := \begin{pmatrix} a \\ b \\ d \end{pmatrix}$. [3]

(b) Check that the vector e_2 so produced is in the plane spanned by v_1 and v_2 . [1]

(c) Find a vector with integer entries which is orthogonal to both e_1 and v_2 . [2]

(d) Use the Gram-Schmidt process with e_1 , e_2 and $v_3 := \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$. Identify the relation between the e_3 produced and the answer to the previous part of the question and explain why. [3]