

Math115 Test 3: Eigenvalues and Powering

February 16th, 2010

Answer all part of the question and give complete reasons and checks for your answers. Attach all rough working and do not erase anything that might earn you marks. The parts of the questions are weighted as shown in square brackets on the right.

1. (a) Find E^n using diagonalisation of E . [8]

$$E := \begin{pmatrix} \frac{19}{3} & \frac{32}{3} \\ -\frac{14}{3} & -\frac{25}{3} \end{pmatrix}$$

- (b) Using algebra, find a 2×2 matrix $F \neq I$ which satisfies $F^2 = I$. [2]

2. We are given this matrix:

$$B := \begin{pmatrix} 2 & 0 & -3 \\ -3 & 5 & 6 \\ 6 & -6 & -10 \end{pmatrix}$$

- (a) Given that B has $\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$ as an eigenvector, find the eigenvalue for it. [1]

- (b) Without doing any row or column operations, expand the determinant of $(B - \lambda I)$ using Laplace to make a cubic polynomial (if necessary) and factor it to get the other two eigenvalues. [5]

- (c) Find one of the other two eigenvectors of B . [4]