Cape Breton University

Math115

MATRIX ALGEBRA

March 2011

Time : $\frac{3}{2}$ hours

Please answer any THREE of these questions, please make sure to give all reasoning and working for all questions answered. Start a fresh sheet of paper for each question attempted.

Q1. (a) Find the inverse of $P := \begin{pmatrix} 9 & -3 & 1 \\ 1 & 1 & 1 \\ 9 & 3 & 1 \end{pmatrix}$. [5]

(b) Use P^{-1} to find an exact fit quadratic which passes through the points (-3,3), (1,7) and (3,9). [3]

(c) Use P^{-1} once more (and the fact that $(X^T)^{-1} = (X^{-1})^T$ for any invertible matrix X) to find all solutions to the recurrence $c_{n+1} := c_n + 9c_{n-1} - 9c_{n-2}$ such that $c_0 = 3$, $c_1 = 2$, $c_2 = 14$. Check your answer for c_3. [4]

Q2. (a) Find the eigenvalues of both of these matrices and check that their eigenvectors are the same. [6]

$$C := \begin{pmatrix} -63 & 143 \\ -30 & 68 \end{pmatrix}, \quad E := \begin{pmatrix} 200 & -429 \\ 90 & -193 \end{pmatrix}$$

(b) Multiply C and E and check that CE has the same eigenvectors and explain how the eigenvalues of a product of such matrices will be related to the eigenvalues of the original matrices and why. [3]

(c) Explain why $XY = YX$ for any matrices which share the same eigenvectors. [3]

Q3. (a) Find all solutions to this system of equations by reducing them to an equivalent of Reduced Row Echelon Form. [10]

$$\begin{aligned}
-v+3\,w+x+3\,z &= 7\\
3\,v+w-x+3\,y+2\,z &= 1\\
4\,v+4\,w+2\,x+3\,y+z &= -2\\
v-w+3\,x+y-z &= 6\\
v+6\,w+3\,x+4\,y+9\,z &= 26
\end{aligned}$$

(b) Use the answer to give a particular solution and a homogeneous solution both consisting only of integers. [2]

Q4. (a) For which value of y is this matrix guaranteed to be non-singular and what is the determinant of J equal to in this case? [4]

$$J := \begin{pmatrix} -2 & x & 3 \\ 0 & -1 & 3 \\ y & 4 & x \end{pmatrix}$$

(b) Why does having two x terms in J not change the way this question works from when we only have one? [1]

(c) What values can the rank of J take? Explain, giving an example of values of x and y and forming a RREF for each of the possible cases. [4]

(d) Give a different 3×3 matrix F with two xs and one y entry which can have a smaller rank than J could ever have for some non-zero values of x and y. How low can the rank be if we additionally insist that each x and y is non-zero and they are all in different rows and columns? [3]

END OF QUESTION PAPER