University College of Cape Breton MATH115

MATRIX ALGEBRA

March 2003 Time : 1.5 hours

All questions carry an equal weight and can be attempted in any order. Clearly write your answers to the questions showing all working, explanation and checks used.

Q1. (a) Calculate the determinant of $A := \begin{bmatrix} -r & -1 & -6 \\ 4 & -2 & r+1 \\ 6 & -3 & 2 \end{bmatrix}$. [8]

(b) Letting $r := 0$ in A, solve the system of equations $[x\ y\ z]A^T = [-2\ 1\ 2]$. [7]

Q2. (a) Verify that $\begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ -1 \\ 1 \\ -1 \end{bmatrix}$ are eigenvectors of $\begin{bmatrix} 3 & 1 & -1 & -2 \\ -4 & -3 & 4 & 4 \\ 6 & 5 & -4 & -6 \\ -4 & -4 & 4 & 5 \end{bmatrix}$. [2]

(b) Use carefully chosen row and column operations to find the other two eigenvectors. [13]

Q3. (a) Find the inverse of $C := \begin{bmatrix} 2 & 4 & -3 \\ 0 & -1 & -2 \\ 0 & 0 & 1 \end{bmatrix}$ [4]

(b) Similarly, show that the inverse of any invertible 3×3 upper triangular matrix will also be upper triangular. [4]

(c) Using the adjoint or by considering the individual elements of $UX = I$, prove that $X := U^{-1}$ will also be an upper triangular matrix if U and X are $n \times n$. [7]

Q4. (a) Diagonalise $F := \begin{bmatrix} -59 & 84 \\ -40 & 57 \end{bmatrix}$ by finding its eigenvectors. [7]

(b) Use the diagonalisation to evaluate F^k for any integer k. [3]

(c) Calculate the inverse of F by substituting $k = -1$ and verify that this inverse formula will always work for any diagonalised matrix. [5]