Math115 Test 1: Matrix multiplication and Inverses

1. (a) We are given these three matrices:

$$E := \begin{pmatrix} -1 & -1 & 1 \end{pmatrix} F := \begin{pmatrix} 2 \\ -3 \end{pmatrix} G := \begin{pmatrix} -1 & 5 & -1 \\ -1 & 11 & -6 \end{pmatrix}$$

   (b) What are these matrices, if they exist? EE^T, FF^T, FG^T, $FE + 2G$?

   (c) What are the ranks of E, F and G and the matrices calculated above?

2. Find the inverse of $H := \begin{bmatrix} 1 & -1 & 1 & -1 \\ 0 & -1 & 0 & -1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & -1 & 0 \end{bmatrix}$?

3. Show that, for any matrix C, CC^T is symmetric.

4. (a) Use row operations to find all solutions to:

$$\begin{aligned} w - 2x - 5y + 4z &= 6 \\ -w + 3x + 8y - 6z &= -5 \\ -2w + 3x + 7y - 6z &= -13 \end{aligned}$$

   (b) Verify that $w = 6$, $x = -3$, $y = 2$, $z = 1$ is a solution to both the initial equations and your final parametrised answer.