

Math115 Test 2: Determinants and Eigenvalues

1. (a) Show that $\begin{pmatrix} 5 \\ -3 \\ 4 \end{pmatrix}$ is an eigenvector of $\begin{bmatrix} -2 & 20 & 20 \\ 4 & -10 & -14 \\ -2 & 14 & 15 \end{bmatrix}$ and deduce its eigenvalue.
(b) Find the other two eigenvalues and one of the other eigenvectors.
2. (a) Show that zero is an eigenvalue of $A := \begin{bmatrix} 1 & -2 \\ -2 & 4 \end{bmatrix}$ and also that $\det(A) = 0$ also.
(b) For a general 2×2 matrix of determinant zero show that zero is an eigenvalue.
(c) Explain why any matrix of determinant zero will have zero as an eigenvalue.
3. (a) Find the inverse of $B := \begin{bmatrix} 3 & 2 & 1 \\ -2 & 0 & 1 \\ 7 & 3 & 0 \end{bmatrix}$ and hence solve this equation: $[x \ y \ z](2B)^T = [16 \ -22 \ 52]$.