Math115 Test 5: Vectors and Orthogonality

Please complete individually and hand in before Monday 7th April at Noon. Any copying will result in those copying and copied getting zero, so please push well under my door if you can't give your paper to me in person!

1. (a) Find the intersection of the plane R which has equation $-7x + 3y + 5z = 11$ with the line $\begin{pmatrix} 5 \\ -2 \\ 8 \end{pmatrix} + \begin{pmatrix} 1 \\ -3 \\ 2 \end{pmatrix} t$.

   (b) What is the equation of the plane S parallel to R which passes through $\begin{pmatrix} -2 \\ 0 \\ -1 \end{pmatrix}$?

   (c) What is the distance between the planes R and S?

2. (a) Given this set of vectors show that they are mutually orthogonal:

$$\begin{pmatrix} 1 \\ 3 \\ -2 \\ 4 \\ -3 \end{pmatrix}, \quad \begin{pmatrix} 5 \\ -2 \\ 5 \\ 2 \\ -1 \end{pmatrix}, \quad \begin{pmatrix} 1 \\ 2 \\ 0 \\ 2 \\ 5 \end{pmatrix}$$

   (b) Use the Gram Schmidt method with $\begin{pmatrix} -6 \\ -5 \\ 3 \\ 4 \\ 5 \end{pmatrix}$ to find a vector which is orthogonal to the three above and is a member of a basis which spans the space of these four vectors.

   (c) Use matrices to find out what linear combination of the basis vectors will give us $\begin{pmatrix} 26 \\ 43 \\ -44 \\ -28 \\ -69 \end{pmatrix}$.

3. None of these are vector spaces. Identify and prove which of the three vector space axioms are true for each set and give counterexamples for those which are false.

$$\{(x, y) \mid (x + y)^2 < 1\}, \quad \{(x, y, z) \mid 3x - 5y + 2z = 0 \text{ but } y \neq 1\}, \quad \{(x, y, z) \mid z \leq 0\},$$