2013 Q1

(a)

$$A = \begin{pmatrix} 25 & -5 & 1 \\ 1 & -1 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \\ 4 & 2 & 1 \\ 9 & 3 & 1 \end{pmatrix} \qquad \underline{v} = \begin{pmatrix} 1 \\ 2 \\ 1 \\ -1 \\ -2 \\ 3 \end{pmatrix}$$

$$A^T A = \begin{pmatrix} 724 & -90 & 40 \\ -90 & 40 & 0 \\ 40 & 0 & 6 \end{pmatrix}$$

$1+1+1+1+1+1=6$

$-5-1+0+1+2+3=-6+6=0$ (crossed out)

$25+1+1+1+4+9=40$

$-125-1+0+1+8+27=-90$

$625+1+0+1+16+81=724$

$$A^T\underline{v} = \begin{pmatrix} 25+2+0+-1-8+27 \\ -5-2+0-1-4+9 \\ 1+2+1-1-2+3 \end{pmatrix} = \begin{pmatrix} 45 \\ -3 \\ 4 \end{pmatrix}$$

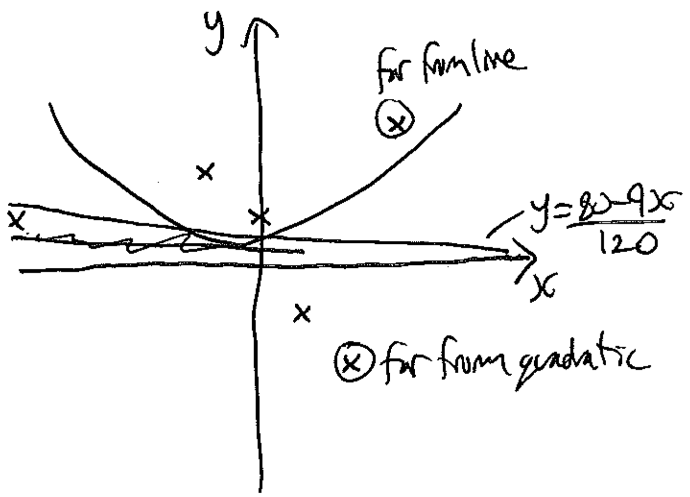
$$\left(\begin{array}{ccc|c} 724 & -90 & 40 & 45 \\ -90 & 40 & 0 & -3 \\ 40 & 0 & 6 & 4 \end{array}\right) \xrightarrow{\text{rowops}} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 5/110 \\ 0 & 1 & 0 & 3/110 \\ 0 & 0 & 1 & 40/110 \end{array}\right)$$

so best fit quadratic is $y = \dfrac{5x^2+3x+40}{110}$

(b)

$$\left(\begin{array}{cc|c} 40 & 0 & -3 \\ 0 & 6 & 4 \end{array}\right)$$ gives directly $40m=-3$, $6b=4$ so $y = \dfrac{2}{3} - \dfrac{3x}{40} = \dfrac{80-9x}{120}$ is best fit straight line

(c)



$\dfrac{dy}{dx} = \dfrac{10x+3}{110}$ so min $x = -\dfrac{3}{10}$

$y = \dfrac{\frac{-45}{100} - \frac{9}{10} + 40}{110} \simeq \dfrac{38}{110}$

$x=3$ $y = \dfrac{80-9x}{120} = \dfrac{53}{120}$ $\quad 3 - \dfrac{53}{120} = \dfrac{307}{120}$

$x=2$ $y = \dfrac{20+6+40}{110} = \dfrac{66}{110}$ $\quad -2 - \dfrac{3}{5} = \dfrac{-13}{5}$

2013 Q2

(a) $B\begin{pmatrix}2\\2\\1\end{pmatrix} = \begin{pmatrix}-36-204+246\\30+150-174\\10+48-55\end{pmatrix} = \begin{pmatrix}6\\6\\3\end{pmatrix} = 3\times\begin{pmatrix}2\\2\\1\end{pmatrix}$ so eigenvalue is 3

$$\det(B-\lambda I) = \det\left(\begin{pmatrix}-18-\lambda & -102 & 246\\15 & 75-\lambda & -174\\5 & 24 & -55-\lambda\end{pmatrix}\right) \overset{R_2 \leftarrow R_2 - 3R_3}{=} \det\left(\begin{pmatrix}-18-\lambda & -102 & 246\\0 & 3-\lambda & -9+3\lambda\\5 & 24 & -55-\lambda\end{pmatrix}\right)$$

$$\overset{C_3 \leftarrow C_3 + 3C_2}{=} \det\left(\begin{pmatrix}-18-\lambda & -102 & -60\\0 & 3-\lambda & 0\\5 & 24 & 17-\lambda\end{pmatrix}\right)$$

$$= (3-\lambda)\times\det\left(\begin{pmatrix}-18-\lambda & -60\\5 & 17-\lambda\end{pmatrix}\right)$$

$$= (3-\lambda)\times\left(\lambda^2+18\lambda-17\lambda-306+300\right)$$

$$= (3-\lambda)\left(\lambda^2+\lambda-6\right) = (3-\lambda)(\lambda+3)(\lambda-2)$$

(b) nondom is 2

$$\left(\begin{array}{ccc|c}-20 & -102 & 246 & 0\\15 & 73 & -174 & 0\\5 & 24 & -57 & 0\end{array}\right) \to \left(\begin{array}{ccc|c}0 & -6 & 18 & 0\\0 & 1 & -3 & 0\\5 & 24 & -57 & 0\end{array}\right) \to \left(\begin{array}{ccc|c}0 & 0 & 0 & 0\\0 & 1 & -3 & 0\\5 & 0 & 15 & 0\end{array}\right)$$

$x = -\frac{15}{5}z = -3z$

$y = 3z$

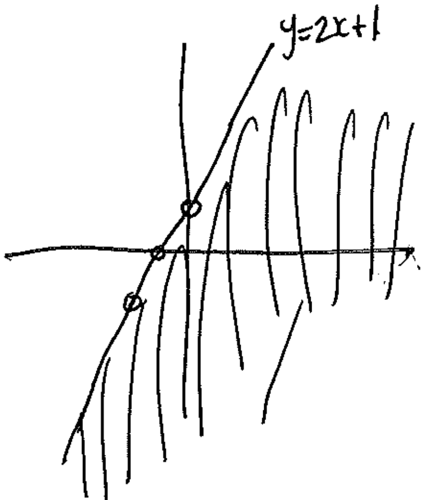
$$\underline{v} = \begin{pmatrix}-3\\3\\1\end{pmatrix}z$$

(c) If $\underline{v}_0 = \begin{pmatrix}-3\\3\\1\end{pmatrix}$ or a multiple $B^k\underline{v}_0 = \begin{pmatrix}-3\times2^k\\3\times2^k\\2^k\end{pmatrix}$

If $\underline{v}_0 =$ general vector then $B^k\underline{v}_0 \to \begin{pmatrix}\sim\times3^k+\sim\times(-3)^k\\\sim\times3^k+\sim\times(-3)^k\\\sim\times3^k+\sim\times(-3)^k\end{pmatrix}$ so depends on k even or odd

Q3 2013

$y \leq 2x+1$



i) 0 is in $x=0$ $y=0$

$0 \leq 2\times0+1=1$ ✓

ii) $\begin{pmatrix}0\\1\end{pmatrix}$ is in $x=0$ $y=1$

$1 \leq 2\times0+1=1$ ✓

$\begin{pmatrix}-1\\-1\end{pmatrix}$ is in $x=-1$ $y=-1$

$-1 \leq 2\times-1+1=-1$ ✓

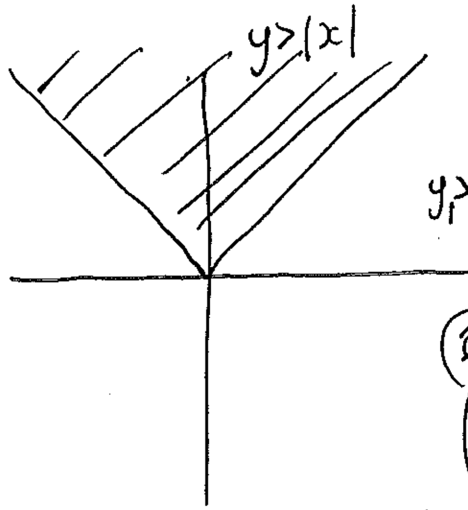
But $\begin{pmatrix}0\\1\end{pmatrix}+\begin{pmatrix}-1\\-1\end{pmatrix}=\begin{pmatrix}-1\\0\end{pmatrix}$ is not

$x=-1$ $y=0$

$0 \nleq 2\times-1+1=-1$

iii) $7\times\begin{pmatrix}0\\1\end{pmatrix}=\begin{pmatrix}0\\7\end{pmatrix}$ But $7 \nleq 2\times0+1=1$

only i) true



$0 \not> 0$ $\begin{pmatrix}0\\0\end{pmatrix}$ not in

$y_1>|x_1|$ $y_2>|x_2|$

$y_1+y_2>|x_1|+|x_2| \geq |x_1+x_2|$

$\begin{pmatrix}0\\1\end{pmatrix}$ is in since $1>|0|=0$

But $-2\times\begin{pmatrix}0\\1\end{pmatrix}$ is $\begin{pmatrix}0\\-2\end{pmatrix}$ which is

not in since $-2 \not> |0|=0$