Math1204 Test 2

February 6^{th} 2013

Answer all questions and give complete reasons and checks for your answers. Please do not erase anything, just put a line through your work and continue; you cannot lose marks for anything you write. The parts of the questions are weighted as shown and can be answered in any order.

1. (a) Use one row or column operation on this matrix M which creates a zero but leaves the determinant unchanged and then use a cofactor expansion to get the determinant in terms of a 3×3 matrix determinant. Evaluate that determinant using a cofactor expansion and hence find the determinant of M. [5]

$$M := \begin{pmatrix} 3 & 1 & -1 & -1 \\ -1 & -1 & 0 & 0 \\ 1 & 2 & 3 & -1 \\ 2 & -1 & 0 & -1 \end{pmatrix}$$

   (b) Evaluate the determinant of N and hence find which value of y guarantees that N is never singular. [3]

$$N := \begin{pmatrix} 2 & y & -2 \\ 1 & 0 & 2 \\ x & 1 & 1 \end{pmatrix}$$

   (c) Create a matrix with no zeroes and an x in one entry which always has determinant 13. Try to at least get one which is never singular for any value of x. [2]

2. (a) Given that $4BX + AB^T = B^{-1}$, solve for X, explaining what rule of algebra you use at each step. Why must A be a square matrix if this equation is to have a unique solution? [5]

   (b) Find X using your solution if $A := \begin{pmatrix} 4 & 3 \\ -5 & -1 \end{pmatrix}$ and $B := \begin{pmatrix} 3 & 7 \\ -2 & -5 \end{pmatrix}$. [5]