Cape Breton University

Math 1204

MATRIX ALGEBRA

April 2015 — Time : 3 hours

Please answer any FIVE of these questions, please make sure to give all reasoning and working for all questions answered. Start a fresh sheet of paper for each question attempted.

Q1. Suppose A^{-1} and B^{-1} exist and let X satisfy: $\left(2B^{-1} + XA\right)^{-1} = \frac{1}{3}A^{-1}B$

(a) Show that if X is all zeros then A must be a multiple of the identity matrix. [4]

(b) Solve the equation for X, giving all algebra rules used for simplification. [5]

(c) If $X = B$ now explain why we could have $A = I$ and if so then $B^2 = I$. Give an example of a B which is not the identity matrix which satisfies this equation. [3]

Q2. Consider the recurrence $c_{k+1} = 2c_k + 9c_{k-1} - 18c_{k-2}$ and $c_0 := 45$, $c_1 := 50$, $c_2 := 190$.

(a) Find c_3 from the recurrence and find the eigenvalues of the underlying matrix by factoring a cubic polynomial. Give their eigenvectors in standard form. [3]

(b) Form P, the matrix of eigenvectors, use the adjoint method to find its inverse and hence give the formula for c_k in terms of powers of its eigenvalues. [7]

(c) Explain what the behaviour of c_k will be as k goes to infinity, why the dominant eigenvalue situation is unusual this time and a set of initial values which would keep decreasing indefinitely but are not just a multiple of a k^{th} power. [2]

Q3. (a) Use row operations to find the dependency equation between these four vectors and check it holds. [8]

$$\underline{v}_1 := \begin{pmatrix} 3 \\ 1 \\ 1 \\ 1 \end{pmatrix}, \quad \underline{v}_2 := \begin{pmatrix} 3 \\ 2 \\ -1 \\ -1 \end{pmatrix}, \quad \underline{v}_3 := \begin{pmatrix} 2 \\ 1 \\ 1 \\ 2 \end{pmatrix}, \quad \underline{v}_4 := \begin{pmatrix} 1 \\ 2 \\ 0 \\ 3 \end{pmatrix}$$

(b) Show that $\underline{n} := \begin{pmatrix} -1 \\ 2 \\ 2 \\ -1 \end{pmatrix}$ is perpendicular to v_1, v_2, v_3 and v_4. If $L := \begin{pmatrix} 5+4t \\ 5-2t \\ 4t-3 \\ t-6 \end{pmatrix}$ is a line, use $\underline{n}$ to find the point where L intersects the vector space containing the vectors from part (a). [4]

Q4. (a) If $M := \begin{pmatrix} -14 & -6 & 9 \\ -72 & -20 & 36 \\ -90 & -30 & 49 \end{pmatrix}$, find and factor the determinant of $M - \lambda I$. [6]

(b) Find two eigenvectors of the repeated eigenvalue, manipulate them to make an orthogonal basis for the eigenvectors and check that $M\underline{v} = \lambda\underline{v}$. [6]

Q5. Data is collected and the following points are found (0,-4), (1,5), (-2,-3), (-3,-1), (4,8).

(a) Find the best fit quadratic by solving the appropriate 3×3 matrix equation. [8]

(b) Plot the points and the quadratic and also calculate the fractional vertical differences between the y values and the curve values. [4]

Q6. (a) Create 3×3 matrices of rank 1, 2 and 3 which only include the numbers +4 and -4, explaining why their ranks are what they are. Multiply them all by the matrix of rank 1 and explain why all of these products also have rank 1. [8]

(b) What are the determinants of your matrices? Explain what all of the possible determinants are for such a rank 3 matrix. [4]

Q7. (a) Find the dot product form of the plane P which is defined by $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ -5 \\ 2 \end{pmatrix} + j \begin{pmatrix} 3 \\ 2 \\ -4 \end{pmatrix} + k \begin{pmatrix} 5 \\ 3 \\ -7 \end{pmatrix}$ and where it intersects with $\begin{pmatrix} x \\ y \\ z \end{pmatrix} \circ \begin{pmatrix} -3 \\ 4 \\ 2 \end{pmatrix} = 12$. [6]

(b) Which of the vector space axioms are true for these two different relations? Draw on different x-y planes the values which satisfy the relations and give counterexamples to show axioms false or explain algebraically why the axioms are true. [6]

i) $xy = 0$, ii) $y > 3x - 1$

END OF QUESTION PAPER