Math1204 Assignment 5: Easter 2016

Answer all questions and give complete reasons and *checks* for your answers, writing words to explain your conclusions. Please do not erase anything, just put a line through your work and continue; just indicate which answer is your final one.

The questions are weighted as shown and can be answered in any order. Because of everyone having different numbers, do not expect nice round numbers to come as your answers. Use proper fractions for your answers and give any direction vectors using integers.

The numbers represented by a, b, c, d and e should be replaced by the largest five digits of your registration number in non-increasing order and any zero should be replaced by -1. For instance, if my registration number was 20106002 then i would take $a = 6$, $b = 2$, $c = 2$, $d = 1$ and $e = -1$.

Your final solutions should be handed in in class on March 31$^{\text{st}}$ if you want them back before the final exam (which is on Monday 11$^{\text{th}}$ starting at 14:00). Otherwise I will hand them back at the exam if they are handed in on April 5$^{\text{th}}$.

1. Let P be the 3-dimensional plane $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 \\ -1 \\ a \end{pmatrix} + j \times \begin{pmatrix} 3 \\ c \\ -5 \end{pmatrix} + k \times \begin{pmatrix} b \\ 1 \\ 2 \end{pmatrix}$ and let L be the line $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} d \\ 3 \\ -2 \end{pmatrix} + t \times \begin{pmatrix} 4 \\ 5 \\ e \end{pmatrix}$.

   (a) Determine the dot product form of P. How can I be sure that your P is not a line? [4]

   (b) Find the point of intersection between P and L. Use row operations to find the values of j and k that give this point. [6]

   (c) Use Gram-Schmidt to get an orthogonal basis for the direction vectors of P then use Gram-Schmidt once more with $\begin{pmatrix} 11 \\ -5 \\ 7 \end{pmatrix}$ and explain why you must get a multiple of a vector you have already calculated. [4]

2. Let these be three different Hyperplanes in 4 dimensions:

$$\begin{pmatrix} w \\ x \\ y \\ z \end{pmatrix} \circ \begin{pmatrix} 3 \\ 2 \\ 1 \\ 4 \end{pmatrix} = a \ , \quad \begin{pmatrix} w \\ x \\ y \\ z \end{pmatrix} \circ \begin{pmatrix} 3 \\ 1 \\ 2 \\ 2 \end{pmatrix} = b \ , \quad \begin{pmatrix} w \\ x \\ y \\ z \end{pmatrix} \circ \begin{pmatrix} 4 \\ 3 \\ 1 \\ 6 \end{pmatrix} = d$$

   (a) Find the intersection of the first two hyperplanes and identify what sort of object it is by its dimension. [3]

   (b) By dimensions, what would you expect the intersection of all three hyperplanes to be? Solve the corresponding matrix equation and hence find out what it is for you. What is it about the hyperplanes that caused this to happen? [3]