Cape Breton University

Math 1204

MATRIX ALGEBRA

February 2016

Time : $\frac{3}{2}$ hours

Please answer any THREE of these questions, please make sure to give all reasoning and working for all questions answered. Start a fresh sheet of paper for each question attempted.

Q1. (a) Verify that $\begin{pmatrix} 1 \\ 5 \\ -2 \end{pmatrix}$ is an eigenvector of $N := \begin{pmatrix} -5 & 2 & 1 \\ 72 & -13 & -4 \\ -216 & 50 & 20 \end{pmatrix}$. [1]

(b) Find the other two integer eigenvalues of N by evaluating the determinant of $(N - \lambda I)$ and using long division on it using the known eigenvalue from (a). [6]

(c) Find one of the other two eigenvectors of N. [4]

Q2. (a) Solve this equation for the unknown matrix X, expanding all possible brackets and simplifying, stating which rules of matrix algebra you are using at each stage. [5]

$$A^T X A^{-1} = (3I + B)A^T$$

(b) Given $A := \begin{pmatrix} 3 & 4 \\ -2 & -3 \end{pmatrix}$ and $B := \begin{pmatrix} 2 & 1 \\ 5 & 0 \end{pmatrix}$, find A^{-1} and A^T and evaluate your answer for X from part (a). [4]

(c) Find all of the 2×2 matrices C with integer values such that $C = C^T = C^{-1}$. [2]

Q3. (a) Evaluate the determinant of $E := \begin{pmatrix} 5 & -1 & -5 \\ -6 & x & 1 \\ 8 & 1 & y \end{pmatrix}$. For which value of x is E guaranteed to be non-singular? What is the determinant of E in this case? What value of y makes E singular when $x = 1$? [5]

(b) Now using $y = -8$, but letting x be unknown, find the inverse of E using the adjoint method. [6]

Q4. (a) Find both eigenvalues and eigenvectors of $F := \begin{pmatrix} 4 & -11 \\ 5 & -12 \end{pmatrix}$. [5]

(b) Verify that both eigenvectors of F are also eigenvectors of F^{-1} but not F^T. Explain why the eigenvalues of any 2×2 matrix M will necessarily be the same as those of M^T, and hence or otherwise find both eigenvectors of F^T. [6]

END OF QUESTION PAPER