1204 Assignment 5 2017

Let $a := 6$ $b := 5$ $c := -1$ $d := -2$

Q1) $P:$ $\begin{pmatrix} x \\ y \\ z \end{pmatrix} \cdot \begin{pmatrix} -12 \\ 2 \\ 5 \end{pmatrix} = -3$ $L:$ $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \\ 6 \end{pmatrix} + t \times \begin{pmatrix} 6 \\ 11 \\ -1 \end{pmatrix}$

(a) $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2+6t \\ -1+11t \\ 6-t \end{pmatrix}$ subs in P $\begin{pmatrix} 2+6t \\ -1+11t \\ 6-t \end{pmatrix} \cdot \begin{pmatrix} -12 \\ 2 \\ 5 \end{pmatrix} = -24-72t-2+22t+30-5t$

$= 4-55t$

Thus t value at intersection is when $4-55t=-3$

eg $t = 7/55$

Point of intersection is $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \\ 6 \end{pmatrix} + \frac{7}{55}\begin{pmatrix} 6 \\ 11 \\ -1 \end{pmatrix} = \frac{1}{55}\begin{pmatrix} 110+42 \\ -55+77 \\ 330-7 \end{pmatrix}$

$= \frac{1}{55}\begin{pmatrix} 152 \\ 22 \\ 323 \end{pmatrix}$

(b) we need solution to

$$\left(\begin{array}{ccc|c} -12 & 2 & 5 & -3 \\ 3 & -4 & 2 & -5 \end{array}\right)$$

$R2 \Leftarrow R2 + 2R1$

$$\left(\begin{array}{ccc|c} -12 & 2 & 5 & -3 \\ -21 & 0 & 12 & -11 \end{array}\right)$$

$R1 \Leftarrow R1 - \frac{4}{7}R_2$

$$\left(\begin{array}{ccc|c} 0 & 2 & -\frac{13}{7} & \frac{23}{7} \\ -21 & 0 & 12 & -11 \end{array}\right)$$

$R_1 \Leftarrow R_1 \times \frac{1}{2}$

$R_2 \Leftarrow R_2 \times -\frac{1}{21}$

$$\left(\begin{array}{ccc|c} 0 & 1 & -\frac{13}{14} & \frac{23}{14} \\ 1 & 0 & -4/7 & \frac{11}{21} \end{array}\right)$$

so $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4/7\,t + \frac{11}{21} \\ 13/14\,t + \frac{23}{14} \\ t \end{pmatrix}$

(b) so $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 8 \\ 13 \\ 14 \end{pmatrix} \frac{t}{14} + \begin{pmatrix} 11/21 \\ 23/14 \\ 0 \end{pmatrix}$

Check $\begin{pmatrix} -12 & 2 & 5 \\ 3 & -4 & 2 \end{pmatrix} \begin{pmatrix} 8 \\ 13 \\ 14 \end{pmatrix} = \begin{pmatrix} -96+26+70 \\ 24-52+28 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ ✓

$\begin{pmatrix} -12 & 2 & 5 \\ 3 & -4 & 2 \end{pmatrix} \frac{1}{42} \begin{pmatrix} 22 \\ 69 \\ 0 \end{pmatrix} = \frac{1}{42} \begin{pmatrix} -264+138 \\ 66-276 \end{pmatrix} = \begin{pmatrix} 3 \\ -5 \end{pmatrix}$ ✓

(c) If they meet then $\begin{pmatrix} 2 \\ -1 \\ 6 \end{pmatrix} + t \times \begin{pmatrix} 6 \\ 11 \\ -1 \end{pmatrix} = \begin{pmatrix} 22/42 \\ 69/42 \\ 0 \end{pmatrix} + \alpha \times \begin{pmatrix} 8 \\ 13 \\ 14 \end{pmatrix}$

so $\left(\begin{array}{cc|c} 6 & -8 & -\frac{62}{42} \\ 11 & -13 & \frac{11}{42} \\ -1 & -14 & -6 \end{array}\right)$ needs to be satisfied for intersection

$R_1 \Leftarrow R_1 + 6R_3 \quad R_2 \Leftarrow R_2 + 11R_3$

$\left(\begin{array}{cc|c} 0 & -92 & \frac{787}{21} \\ 0 & -167 & -\frac{887}{14} \\ -1 & -14 & -6 \end{array}\right)$ and $R_2 \Leftarrow R_2 - \frac{167}{92} R_1$ $\left(\begin{array}{cc|c} 0 & -92 & -\frac{787}{21} \\ 0 & 0 & \frac{1289}{276} \\ -1 & -14 & -6 \end{array}\right)$

since $0 \neq \frac{1289}{276}$ the lines do not intersect

(d) For P we need two vectors perpendicular to $\begin{pmatrix} -12 \\ 2 \\ 5 \end{pmatrix} = \underline{n}$

Let's say $\underline{v}_1 = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$ and $\underline{v}_2 = \begin{pmatrix} 0 \\ 5 \\ -2 \end{pmatrix}$

$\underline{v}_1 \cdot \underline{n} = -12+2+10 = 0$

$\underline{v}_2 \cdot \underline{n} = 0+10-10 = 0$

$\underline{e}_1 = \underline{v}_1 = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$ $\quad \underline{e}_2 : \begin{pmatrix} 0 \\ 5 \\ -2 \end{pmatrix} - \frac{\begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 5 \\ -2 \end{pmatrix}}{\begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}} \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 0 \\ 5 \\ -2 \end{pmatrix} - \frac{0+5-4}{1+1+4} \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} = \frac{1}{6} \left(\begin{pmatrix} 0 \\ 30 \\ -12 \end{pmatrix} - \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} \right)$

Take $\underline{e}_2 = \begin{pmatrix} -1 \\ 29 \\ -14 \end{pmatrix}$ check $\underline{e}_1 \cdot \underline{e}_2 = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ 29 \\ -14 \end{pmatrix} = -1+29-28=0$ ✓

$\underline{e}_2 \cdot \underline{n} = \begin{pmatrix} -1 \\ 29 \\ -14 \end{pmatrix} \cdot \begin{pmatrix} -12 \\ 2 \\ 5 \end{pmatrix} = 12+58-70=0$

(e) We want to solve $(-12 \; 2 \; 5 \; ; -3)$ so let $x = s$ $z = t$
and $2y = +12s - 5t - 3$
$y = 6s - \frac{5}{2}t - \frac{3}{2}$

That is $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 6 \\ 0 \end{pmatrix} s + \begin{pmatrix} 0 \\ -5/2 \\ 1 \end{pmatrix} t + \begin{pmatrix} 0 \\ -3/2 \\ 0 \end{pmatrix}$

The plane doesn't pass through the origin $\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$ since $\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} -12 \\ 2 \\ 5 \end{pmatrix} = 0 \neq -3$

so we need at least one nonzero in the point.

Similarly, there are no zeros in $\begin{pmatrix} -12 \\ 2 \\ 5 \end{pmatrix}$ so need at least two non zeroes in any direction vector.

Hence the 4 zeros we have is the maximum.

Q2) $\begin{pmatrix} w \\ x \\ y \\ z \end{pmatrix} \cdot \begin{pmatrix} 6 \\ -1 \\ -1 \\ 5 \end{pmatrix} = 2$

(a) For a line to never intersect H it must have direction in H.
eg $\begin{pmatrix} 1 \\ 1 \\ 0 \\ -1 \end{pmatrix}$ since $\begin{pmatrix} 1 \\ 1 \\ 0 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 6 \\ -1 \\ -1 \\ 5 \end{pmatrix} = 6-1+0-5=0$

Just need to pick a point not going to give 2, say $\begin{pmatrix} 0 \\ 3 \\ 0 \\ 0 \end{pmatrix}$

line is $\begin{pmatrix} 1 \\ 1 \\ 0 \\ -1 \end{pmatrix} \times \alpha + \begin{pmatrix} 0 \\ 3 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} \alpha \\ \alpha+3 \\ 0 \\ -\alpha \end{pmatrix}$ and $6\alpha - \alpha - 3 - 0 - 5\alpha = 2$
says $-3 = 2$ no solution in α

Q2)(b) Similarly to (a) we need two directions in H and now a point in H

eg $\begin{pmatrix}1\\1\\0\\-1\end{pmatrix}$ and $\begin{pmatrix}1\\0\\6\\0\end{pmatrix}$ since $6+0-6+0=0$

and $\begin{pmatrix}0\\0\\3\\1\end{pmatrix}$ since $\begin{pmatrix}0\\0\\3\\1\end{pmatrix}\cdot\begin{pmatrix}6\\-1\\-1\\5\end{pmatrix} = 0+0-3+5=2$

Thus a plane completely in H is

$$\begin{pmatrix}w\\x\\y\\z\end{pmatrix} = \begin{pmatrix}1\\1\\0\\-1\end{pmatrix}\times\beta + \begin{pmatrix}1\\0\\6\\0\end{pmatrix}\times\gamma + \begin{pmatrix}0\\0\\3\\1\end{pmatrix}$$

$$= \begin{pmatrix}\beta+\gamma\\ \beta\\ 6\gamma+3\\ 1-\beta\end{pmatrix}$$

check: $\begin{pmatrix}\beta+\gamma\\ \beta\\ 6\gamma+3\\ 1-\beta\end{pmatrix}\cdot\begin{pmatrix}6\\-1\\-1\\5\end{pmatrix} = 6\beta+6\gamma-\beta-6\gamma-3+5-5\beta$

$= 0\beta+0\gamma+2$

$= 2$