Math1204 Test 4

March 20$^{\text{th}}$, 2017

Answer all questions and give complete reasons and checks for your answers. Please do not erase anything, just put a line through your work and continue; you cannot lose marks for anything you write. The parts of the questions are weighted as shown and can be answered in any order.

1. Consider this list of five data points: [(1, 13), (2, 10), (4, 11), (5, 5), (8, 6)].
   (a) Find the least squares best fit straight line $f(x)$ for these data points. [7]
   (b) Calculate the vertical differences from your line to each of the points, check if these differences sum to zero and indicate if you think your $f(x)$ is correct. [2]
   (c) For which value of x does your best fit line predict that $f(x)$ will go below 3? [1]

2. (a) Verify that 2 is a root of the polynomial $x^3 - x^2 - 14x + 24$ and use division to get the other two roots. [1]
   (b) Use the roots (ask me for them if you can't get the correct integers for part (a)) to form and give the diagonalisation of the matrix underlying the recurrence:
   $$b_{n+1} := b_n + 14b_{n-1} - 24b_{n-2} \quad \text{where } b_0 = 51,\ b_1 = 73 \text{ and } b_2 = 185.$$
   Use a method from the course to find a formula that gives b_k for any $k \geq 3$. [7]
   (c) Use your knowledge of dominant eigenvalues to determine, if $b_0 = 2$ and $b_1 = 13$, what unique value of b_2 would ensure that the solution to the recurrence is positive for every $k \geq 0$. [2]