Math 115 Midterm , February 27 2002

Q1: Calculate the inverse of this matrix $B := \begin{bmatrix} 5 & 5 & 4 \\ 5 & 4 & 3 \\ 1 & 2 & 4 \end{bmatrix}$ in *one* of these two ways:

- Form the adjoint of B and find the determinant of B
  or
- Form the 3x6 matrix $(B : I)$ and use row operations to take it to $(I : B^{-1})$.

Prove that the inverse of the matrix XY^{-1} is YX^{-1}.
Simplify this expression as much as possible: $X^{-1}Y^2(XY^2)^{-1}X^3$.

Q2: Show that this matrix $C := \begin{bmatrix} 2 & -11 & 24 & -34 & 37 \\ 6 & 5 & 8 & -2 & 5 \\ -1 & -4 & 4 & -8 & 8 \\ 9 & 17 & -4 & 22 & -19 \end{bmatrix}$ has rank 2. Considering the matrix equation $CX = D$ show that if $D := D_1 = [3, 2, -1, 2]^T$ then X has no possible solutions and list all solutions for X if $D := D_2 = [-4, 28, -8, 52]^T$.

If X and Y are matrices with rank r is it true that $X + Y$ also has rank r? Give reasons and/or counterexamples. What can we say about the rank of XY?

Q3: Using row and/or column operations, and Laplace expansions, evaluate the determinant of this matrix:

$$A := \begin{bmatrix} -3 & -2 & 3 & -1 \\ y & -3 & 2 & 3 \\ -1 & x & 1 & -3 \\ 2 & 3 & -2 & -2 \end{bmatrix}$$

For which values of x and y is this matrix invertible? Using the rules of determinants prove that $\det(A) = \det(D)$ for any matrices A and D related by $AP = PD$ where P is an invertible matrix.

Take home question: Please use Maple if you like but do not copy!
Find a matrix A with integer entries and no zeroes which can be diagonalised to

$$D := \begin{bmatrix} a & 0 & 0 & 0 \\ 0 & b & 0 & 0 \\ 0 & 0 & c & 0 \\ 0 & 0 & 0 & d \end{bmatrix}$$

where a, b, c and d are the last 4 non zero entries of your student ID.
Verify that $A = PDP^{-1}$ for some P.