University College of Cape Breton

MATH115

Introduction to Matrix Algebra

June 2002
Time : 3 hours

Student Name:
Registration Number:

All questions carry an equal weight and can be attempted in any order. Clearly write your answers to the questions showing all working and checks and indicate what each mathematical calculation is doing. The best SIX answers will be counted.

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9

Q1. Using these two matrices A and B, calculate the following matrices, or explain why they are not defined: A^3, $(A+B^T)^{-1}$, $(BA)^2$, $(A^T-2B)^T$, $\det(A)$, $(AB)^{-1}$

$$A := \begin{bmatrix} 2 & -1 & 5 \\ -4 & 2 & -10 \end{bmatrix} \quad B := \begin{bmatrix} 7 & -3 \\ 4 & -1 \\ 0 & 2 \end{bmatrix}$$

Q2. Two sequences are related by $a_{k+1} = 3a_k - b_k$ and $b_{k+1} = 2b_k - 2a_k$.

(a) Explain how the matrix $C := \begin{bmatrix} 3 & -1 \\ -2 & 2 \end{bmatrix}$ represents the two sequences.

(b) Find the eigenvectors and eigenvalues of C and hence diagonalise it.

(c) If $a_0 = 5$ and $b_0 = -8$, find the general value of a_k.

Q3. (a) Find all possible solutions to $AX = B$ with the matrices below, verifying at each stage that $[-9\ \ 5\ \ 8\ \ 4\ \ -3]$ is a part of your solution set.

$$A := \begin{bmatrix} -1 & -3 & 5 & -3 & 1 \\ 3 & 5 & -4 & 4 & -1 \\ 3 & -5 & 5 & 2 & -4 \\ -2 & 4 & 1 & -4 & 5 \end{bmatrix} \qquad B := \begin{bmatrix} 19 \\ -15 \\ 8 \\ 15 \end{bmatrix}$$

(b) If A had rank 2 instead, how many variables would be involved in the general solution? Give an example of such an A and B, proving the rank of A.

Q4. (a) Use row and/or column operations to find the other eigenvalues of this matrix given that two of them are 3 and 1.

$$\begin{bmatrix} -2 & -7 & -1 & -6 \\ -15 & -36 & -7 & -12 \\ 75 & 177 & 34 & 66 \\ 5 & 7 & 1 & 9 \end{bmatrix}$$

(b) Find both eigenvectors corresponding to the eigenvalue 3.

Q5. Which of these sets of vectors are orthogonal? If they are not, find a spanning set of orthogonal vectors covering the same space. Prove that all of the original vectors lie within the spanning space specified.

(a) [4, 1, -1], [-1, 8, 4] , [52, -65, 143]

(b) [5, -3, 2, 1], [-5, 7, -4, -3]

(c) [-2, -4, 3], [5, -2, 1], [1, -10, 7]

Q6. Find the eigenvalues and eigenvectors of

$$G := \begin{bmatrix} -458 & -210 \\ 1001 & 459 \end{bmatrix} \text{ and } H := \begin{bmatrix} -563 & -630 \\ 504 & 564 \end{bmatrix},$$

diagonalise, and hence find a P such that G and H are $H = P^{-1}GP$.

Q7. For each of these sets of vectors, prove whether or not they form a vector space. Draw diagrams and explain which of the three rules they do or do not satisfy and why.

(a) $\{(x, y) : x \geq y \text{ and } y \geq 0\}$

(b) $\{(x, y) : x \leq y + 1 \text{ and } x \geq y - 1\}$

(c) $\{(x, y) : x \neq y\}$

Q8. Given this matrix M, find both its inverse and its LU-factorisation. Check your answers and then give the inverses of L and U and show how they are related to M^{-1}.

$$M := \begin{bmatrix} -1 & 4 & -4 \\ 1 & -3 & 1 \\ 1 & -2 & 0 \end{bmatrix}$$

Q9. (a) Use the general 2x2 inverse formula to show that the inverse of *any* invertible 2×2 lower triangular matrix is also lower triangular.

(b) Use the adjoint formula to prove that this idea is also true for 3×3 matrices.

(c) Give a proof for $n \times n$ matrices using matrix multiplication or otherwise.

END OF QUESTION PAPER