Math 115 Test 1, January 29 2002

Instructions: Each question is worth an equal amount of marks, answer all questions (in any order). Start a new page for each question and write your name and student number upon each sheet handed in. Every form of cheating is prohibited and will be punished by a mark of zero for both parties involved, so it is your responsibility to make sure no-one can see your work.

Q1: Given these four matrices, solve these equations for X or explain why they cannot be solved:

$$A := \begin{bmatrix} 3 & -1 & 0 \\ 1 & 2 & -5 \end{bmatrix} B := \begin{bmatrix} 5 & 1 \\ -4 & -1 \end{bmatrix} C := \begin{bmatrix} 4 & 1 \\ 7 & -3 \\ -1 & -4 \end{bmatrix} D := \begin{bmatrix} -1 & 1 \\ 2 & -2 \end{bmatrix}$$

$$X = AC \quad X^T = 3D - \frac{B}{2} \quad D^T X = A \quad BA = XC \quad A = BX + C^T$$

Q2: *Diagonal matrices* are defined by $[d_{i,j}] = 0$ if $i \neq j$. For which 2x2, 2x3 and 3x3 matrices X does the relationship $DX = XD$ hold? Given two nxn diagonal matrices C and D, prove that $C + D$, CD, C^T and D^{-1} are also diagonal.

Q3: Use row operations to find the general solution to this set of equations:

$$\begin{aligned} -5x_1 + 7x_2 - 2x_3 - 10x_4 + 10x_5 &= 3 \\ 9x_1 - 9x_2 + 4x_3 - 8x_5 &= 5 \\ -5x_1 + +4x_2 + 7x_3 - 9x_4 + 5x_5 &= 1 \\ 7x_1 - 8x_2 + 3x_3 + 5x_4 - 9x_5 &= 1 \end{aligned}$$

Q4: List the row operations needed to take this matrix J to reduced row echelon form, and then make another list, this time of the operations required to find and solve the LU decomposition. Using either set of operations, solve $JX = K$

$$J := \begin{bmatrix} -2 & 4 & 0 \\ 2 & 3 & 1 \\ 3 & 5 & -1 \end{bmatrix} \quad K := \begin{bmatrix} -6 \\ -5 \\ 2 \end{bmatrix}$$

Q2*: For bonus marks, and only when all parts of all other questions have been attempted, explain the general solution for an mxn matrix X for which $DX = XD$, and explain which of the four expressions $C + D$, CD, C^T and D^{-1} are still diagonal for general matrices C and D.