**Math 115 Test 2, February 6 2002**

**Instructions:** Each question is worth an equal amount of marks, answer all questions (in any order). Start a new page for each question and write your name and student number upon each sheet handed in. Every form of cheating is prohibited and will be punished by a mark of zero for both parties involved, so it is your responsibility to make sure no-one can see your work.

**Q1:** Suppose we have a matrix of this form:

$$D := \begin{bmatrix} -1 & x & 2 \\ x & 3 & -2 \\ -2 & 1 & 1 \end{bmatrix}$$

Using row operations, find both values of $x$ which make this matrix become non-invertible.

Using $x := -1$, find the general solution to $DY = \begin{bmatrix} -9 \\ 11 \\ -3 \end{bmatrix}$.

**Q2:** Recall that a matrix is called *symmetric* if $A^T = A$.
Prove that for any two symmetric matrices $F$ and $G$ that these 5 matrices are also symmetric:

$$F^{-1} \quad G^T \quad FF^T \quad F+G \quad F^k$$

For each matrix, clearly indicate which matrix properties you use at each stage.

**Q3:** For these statements, either prove them, or give a counterexample to disprove them. Explain why your counterexample was chosen and give the correct version of the particular statement.

1. $(A+B)^2 = A^2 + 2AB + B^2$
2. If $A^2$ exists then $A$ must be a square matrix
3. If $AC = CB$ then $A$ and $B$ cannot be square matrices

Let us suppose that $E^2 = E$ for some 2x2 matrix $E$ which is neither the identity or the zero matrix. Show that $E$ cannot have an inverse and give three matrices which could be $E$, one of which should have some negative entries.

**Take home question:** Please confer but do not copy!
This equation does have a solution for a 2x2 matrix X. Find it, and investigate why. Using the knowledge thus gained, find a set of matrices which satisfy $AB = CD$ where A is 4x3, B is 3x3, C is 4x2 and D is 2x3 where the (1,2)th element of B is $x$ and the (3,2)th element of B is $y$ where $x$ and $y$ are the last two digits of your registration number.

$$\begin{bmatrix} 8 & 5 \\ 5 & 4 \\ 6 & 2 \end{bmatrix} X = \begin{bmatrix} 3 & -2 \\ 1 & -3 \\ 4 & 2 \end{bmatrix} \begin{bmatrix} 4 & 1 \\ -1 & 0 \end{bmatrix}$$