Math 115 Test 3 , March 13th 2002

Q1: Find the eigenvalues and eigenvectors of this matrix

$$\begin{bmatrix} -7/2 & -1 & -1/2 \\ -1 & -2 & 1 \\ -1/2 & 1 & -7/2 \end{bmatrix}$$

Q2: Verify that the eigenvectors of the matrix $M := \begin{bmatrix} 1/3 & 1/12 \\ -4/3 & 7/6 \end{bmatrix}$ are $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 8 \end{bmatrix}$ and deduce the eigenvalues of M. Identify the dominant eigenvalue and hence, or otherwise, find the general formula for c_n and r_n which are defined by the matrix equation

$$\begin{bmatrix} c_k \\ r_k \end{bmatrix} = M \begin{bmatrix} c_{k-1} \\ r_{k-1} \end{bmatrix} \text{ where } \begin{bmatrix} c_0 \\ r_0 \end{bmatrix} = \begin{bmatrix} 36 \\ 600 \end{bmatrix}$$

Q3: Show that $S := \{(a, 3b - 2a, b)\}$ satisfies all of the conditions required for a subspace. Which of the three subspace conditions do these sets satisfy? Give proofs and/or counterexamples as necessary.

$$\{(x,y) : xy > 0\},\ \{(s,t) : s^2 + t^2 < 1\},\ \{(i + \frac{1}{2}, j + \frac{1}{2}) : i, j \text{ integers}\}$$