Math 115 Test 3 , March 27th 2002

Q1: Verify that $C_1 := \begin{bmatrix} 1 \\ -1 \\ 4 \end{bmatrix}$ and $C_2 := \begin{bmatrix} 2 \\ -2 \\ -1 \end{bmatrix}$ are orthogonal vectors and that they are both eigenvectors of $A := \frac{1}{3}\begin{bmatrix} 10 & 11 & 4 \\ 11 & 10 & -4 \\ 4 & -4 & 13 \end{bmatrix}$. Show that $X := \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ is independent with C_1 and C_2 and hence show how to find a third orthogonal vector C_3 using the Gram-Schmidt process or otherwise. Verify that C_3 is in fact the third eigenvector and list the three eigenvalues.

Find the orthonormal vectors corresponding to these eigenvectors and hence find a matrix Q such that $A = QDQ^T$ with D a diagonal matrix made up of the eigenvalues. Use this to find A^3 and A^{-1}.

Q2: Use the Gram-Schmidt process to find the QR decomposition for this matrix where $QR = B := \begin{bmatrix} 2 & -5 \\ 1 & -2 \\ -2 & 3 \end{bmatrix}$ and Q is a 3x2 orthogonal matrix and R is a 2x2 upper triangular matrix. Verify that $B^TB = R^TR$ for your matrices B and R. Explain why $BZ = Y := \begin{bmatrix} 7 \\ -3 \\ -2 \end{bmatrix}$ has no solution for Z and then solve the matrix equation $(B^TB)Z = B^TY$ and evaluate BZ to see how close this "least squares solution" is to Y.

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