Cape Breton University

MATH205

Discrete Mathematics

9th December 2006

Time : 3 hours

Clearly write your answers to the questions showing all working and checks and indicate what each mathematical calculation is doing. The best SIX answers will be counted.

Q1. We have 5 different types of fruit; 2 grapefruit, 7 apples, 4 bananas, 3 pears and a pineapple.

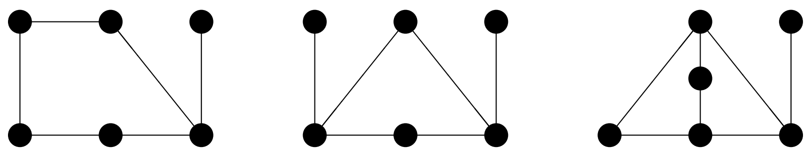
(a) How many ways are there to make a group of 3 different fruits? [2]

(b) How many ways to choose a set of 4 fruits, not necessarily different? [4]

(c) List logically all ways that a group of 7 pieces of fruit can be chosen in such a way that no fruit appears more than twice. [2]

(d) Three people pre-select a favourite and least favourite fruit. If they eat an example of their favourite and destroy one they don't like, how many ways can this happen without someone being unable to take or destroy their selection? [4]

Q2. (a) Given these three graphs identify their chromatic numbers and explain why fewer colours cannot be used to properly colour them. [3]



(b) Find a graph with 6 vertices which requires 4 colours but can be coloured with 3 colours if any vertex is removed. [2]

(c) What is the fewest number of edges that can be added to make them Eulerian? [4]

(d) Find Hamiltonian cycles in each of the above graphs after adding one carefully chosen edge to them or explain why it cannot be done. [3]

Q3. (a) Prove by contradiction that given two real numbers whose product is at most 13 then one of them is less than 4, [6]

(b) What is the largest number n for which it is true that given any two integers whose product is at most n then one of then is at most 4? [2]

(c) Prove by the direct method that the intersection of two sets of cardinality k has maximum cardinality k and the union of them has cardinality at most $2k$. [4]

Q4. We are given two relations using the sets $X := \{a, b, c, d\}$ and $Y := \{e, f, g\}$ as follows:

$$R := \{(a, e), (b, e), (c, f), (d, g)\} \qquad S := \{(e, c), (f, a), (f, d), (g, b)\}$$

(a) Find the set form of the relations $R \circ S$, $S \circ R$ and R^{-1}. [5]

(b) Show that $S \circ (R \circ S) = (S \circ R) \circ S$ for our relations. [2]

(c) Determine whether S and $R \circ S$ are one-to-one, everywhere defined or transitive. [5]

Q5. Prove by induction that [12]

$$\sum_{i=1}^{n} i^4 = \frac{n\,(2\,n+1)\,(n+1)\,(3\,n^2+3\,n-1)}{30}$$

Q6. (a) Solve this recurrence given that $a_1 = -3.9$ and $a_0 = 13$. [8]

$$10a_{n+1} = 3a_n + 4a_{n-1}$$

(b) Use strong induction and the recurrence to prove that a_n has at most n decimal places when written as a real number. [4]

Q7. (a) Simplify $(q \to ((\sim p) \wedge q)) \to p$ and check your result with a truth table. [8]

(b) Given $p(x) :\equiv$ " $|2x - 2| > 3$" and $q(x) :\equiv$ " $\pi - x \geq 1$", prove that $\forall x \in \mathbb{Z}\ (p(x) \vee q(x))$ is true. [4]

Q8. (a) Using the universal set $\mathcal{U} := \{b, d, f, g, h, j, m, p, q, r, s, t, v\}$ determine the contents of these sets and draw a Venn diagram of them: [4]

$$\begin{aligned} A &:= \{\text{letters which follow a vowel in the alphabet}\} \\ B &:= \{\text{letters with circles in their lower case version}\} \\ C &:= \{\text{letters which are beyond n in the alphabet}\} \end{aligned}$$

(b) Identify the portion of the Venn Diagram which is $(\overline{A} \cap B) \cup ((A \cup B) \cap C)$ and explain why it will be a subset of $B \cup C$ in general. [3]

(c) Verify that $|B \cup C| = |B| + |C| - |B \cap C|$ for our sets. [2]

(d) Prove, contrapositively, that if $X \subseteq Y$ then $|X| \leq |Y|$ for any sets X and Y. [3]

END OF QUESTION PAPER