

Math205 Test 2

October 19, 2006

Answer all questions and give complete reasons and checks for your answers. The parts of the questions are weighted as shown and the questions can be answered in any order. Start a fresh side of paper for each question.

1. (a) Simplify this logical statement, and then negate the simplified version. [4]

$$\forall x \in \mathbb{Z}; ((p(x) \vee (\sim (q(x)))) \rightarrow q(x))$$

- (b) Is the above statement true for these two statements? [1]

$$p(x) :\equiv "x^2 < 9"$$

$$q(x) :\equiv "|x - 2| \geq 1"$$

- (c) Given the particular $p(x)$ and $q(x)$ in (b) determine whether or not the following statements are true or false. [5]

$$\exists k \in \mathbb{R}; (p(k) \wedge q(k))$$

$$\forall t \in \mathbb{Q}; ((\sim q(t)) \rightarrow p(t))$$

$$(\exists x \in \mathbb{Z}; (((\sim p(x)) \vee q(x)))) \wedge (\exists x \in \mathbb{Z}; (((\sim q(x)) \vee p(x))))$$

2. (a) Prove that this statement is true by using the method of proof by contradiction. "The product of an even number and an odd number is even" [8]
- (b) Give an example which disproves the statement "If the product of two numbers is even then one of the numbers is odd and the other number is even" and identify this statement in terms of the logical statements used in part (a). [2]