

# Math205 Test 3

November 16, 2006

Answer all questions and give complete reasons and checks for your answers. The parts of the questions are weighted as shown and the questions can be answered in any order. Start a fresh side of paper for each question.

1. (a) Solve this recurrence relation given that  $a_0 = 8$  and  $a_2 = 7$ . [7]

$$a_{n+1} = a_n + 6a_{n-1}$$

- (b) Prove, using induction, that  $a_n < 200(1 - 10n)$  for all  $n$  at least 8. [5]
2. (a) How many different numbers in the range 1 to 13 do you have to choose before some pair are guaranteed to differ by 2? [4]
- (b) Give an example of a set of size one less than your answer in which no pair differs by 2 and a set of size smaller than that to which no new number can be included without making a pair differing by 2. [2]
- (c) Use the patterns above to predict how many would be needed for a difference of 2 *or less* between any pair of numbers and prove this result. [2]