Math205 Test 4: Counting and Relations

November 16, 2006

Answer all questions and give complete reasons and checks for your answers. The parts of the questions are weighted as shown and the questions can be answered in any order. Start a fresh side of paper for each question. Hand in your rough working together with your final answers. You are reminded that plagiarism is a serious offense and if caught you will suffer the penalties specified by the University.

1. The weather and wind speed are recorded for 13 days and for each day the weather is categorised as either sunny, cloudy, rainy or snowy and the wind is high, medium or low.

   (a) Explain using the pigeonhole principle how many times some combination of weather and wind must occur during our record. [1]

   (b) For the first 4 days, how many ways can the wind have been reported? How would your answer change if you were told additionally that no two consecutive days had the same speed? [3]

   (c) Given that during another period of 5 days that at most 2 of them were judged as cloudy, how many ways can this period have been recorded? [4]

   (d) Assuming no combination of weather and wind occured twice, count how many ways that the last 7 days could have been made up as a whole. [2]

   (e) For 3 of the days I was planning to be on holiday, and so didn't want it to be either rainy or snowy. How many different sets of weather/wind would I have been happy with? [2]

   For any of your answers which are below 30, list logically all of the members of your solution set and hence check your answer.

2. We define a relation from the set $S := \{a, b, c, d, e\}$ to the set $T := \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ by letting each member of S in order be related to the last 5 digits of your student ID number so that there are exactly five relations.

   (a) Prove or disprove each of the 4 basic relation properties for your relation. [2]

   (b) Explain why no student could have an onto relation and why every student will have a uniquely defined relation. Will the answers (True or False) of part (a) be the same for every student? [3]

   (c) Give an example of a relation from S to $Q := \{u, v, w, x, y, z\}$ which is everywhere defined and one-to-one but not onto. Prove that if Q was a smaller set no such relation could exist. What if Q was smaller but the relation was now not one-to-one? [3]