# Math205 Test 3

November 20th, 2007

Answer all questions and give complete reasons and checks for your answers. The parts of the questions are weighted as shown and the questions can be answered in any order. Please start a fresh side of paper for each question.

1. (a) Prove by induction that $a_i = \frac{1}{2}(5^{i-1} - 3^i)$ is the solution to $a_{n+1} := 8a_n - 15a_{n-1}$ if $a_1 := -1$ and $a_2 := -2$. [6]

2. (a) Given a set of 27 different numbers between 15 and 60, show by the pigeonhole principle that some pair must differ by exactly 6. [3]

   (b) Suppose you want no difference of 7 to occur, how many different numbers in the same range could you have as a maximum? Give an example. [2]

   (c) If you were now told additionally that the different numbers were all odd, how would that change your answer to both of the two preceding parts? [2]

3. You are given a set of 12 coloured pencils, all different colours.

   (a) How many ways are there for 5 pencils to have been selected as a group? [2]

   (b) One person splits the pencils into sets of three pencils each, then 7 people vote on their favourite set. How many different collections of votes could there be? [3]

   (c) Three of the pencils are blunt. List logically all the ways that a sequence of 4 pencils could be selected with respect to bluntness, ignoring colour. [2]