Math205 Test 4

December 2007

Answer all questions and give complete reasons and checks for your answers. The parts of the questions are weighted as shown and the questions can be answered in any order. Please do not erase any working and hand in your rough work too. You are reminded that plagiarism is a serious offense and can result in academic penalties.

1. (a) Prove by the contrapositive method that if $|X| > |Y|$ then any relation R from X to Y is either not a function or not 1-1. [5]

   (b) Give an example of a relation from $\{a, b, c, d, e\}$ to $\{1, 2, 3, 4\}$ which is not uniquely defined and not 1-1, but which is both everywhere defined and onto. [2]

2. Count all equivalence relations on a four element set and list them logically. [5]

3. Given the graph on your slip of paper answer these questions:

   (a) What are the valencies of all of your vertices? [1]

   (b) Draw a graph with the same valency sequence which is not isomorphic and explain why the two are different. [2]

   (c) Find a Hamiltonian cycle in your graph. [1]

   (d) What is the smallest number of edges which must be added to make your graph Eulerian? Add such a set and describe an Eulerian cycle in the new graph. [2]

   (e) Colour your graph with 3 colours and explain why it cannot be done with 2. [2]