University College of Cape Breton

Math205

DISCRETE MATH

December 2009 Time : 3 hours

Answer FIVE of the SEVEN questions, giving all working and reasoning.

Q1. (a) Prove by induction (or by combining formulae from the handout) that [7]

$$\sum_{j=1}^{n} \left(3^{j-1} - 4j\right) = \frac{3^n - (2n+1)^2}{2}$$

(b) Find from this formula when this sum is positive and prove by induction that it is positive for all integers after that point. [5]

Q2. (a) Use Venn Diagrams to show that $(B \triangle A) \triangle A^c = B^c$. [2]

(b) Expand the $\triangle$s in $(B \triangle A) \triangle A^c$ and use De Morgan's laws but do not simplify further. [3]

(c) Use the associativity of $\triangle$ and set algebra rules to verify the formula. [5]

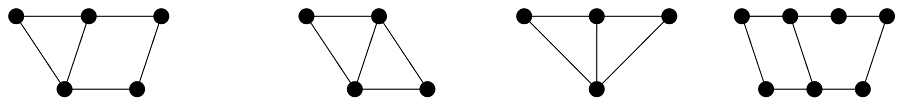
(d) If $|A| = 7$, $|B| = 11$ and $|A \cup B| = 13$ deduce the value of $|A \triangle B|$. [2]

Q3. A graph K is defined as a subgraph of a graph L, written $K \subseteq L$, if K can be formed by deleting vertices and/or edges from L.

(a) Given these edges that make up G, draw it and find its valency sequence. Which edges must be deleted to form a Hamiltonian cycle subgraph? [3]

$$E(G) := \{ae, ag, ah, bd, bf, bh, ce, cg, df, ef, gh\}$$

(b) Determine whether or not these graphs are subgraphs of G. [3]



(c) Explain why the relation "is a subgraph of" is a partial order for graphs. [6]

Q4. (a) Use truth tables to show that $\rightarrow$ is not an associative operator and that $(p \rightarrow q) \vee (r \rightarrow p)$ is a tautology. [5]

(b) Use logic algebra to simplify this proposition by removing all $\sim$ symbols: [6]

$$\sim (\forall y \in \mathbb{R}; \exists x \in \mathbb{Z}; (p(x,y) \rightarrow (\sim q(x,y))) \wedge (\sim ((\sim p(x,y)) \wedge q(x,y))))$$

(c) Give an example of functions for which your simplified statement is true. [1]

Q5. There are 10 entrants in a table tennis competition, 3 of them are classed by age as veterans, 2 are seniors and the remainder are juniors. All entrants play one match against every other entrant and, after all games have been played, they are arranged in an order without any ties.

(a) How many possible arrangements are there for the top two and bottom two ranked players when only considering the age groups of the players? [4]

(b) One particular family contains a veteran, a senior and a junior and they manage to earn the top three places. How many other ways could the top three group have been made up? [3]

(c) How many games are played in total? List all of the games that will be played between the juniors. [3]

(d) What percentage of games involved at least one veteran? [2]

Q6. (a) Determine carefully, giving your reasons, the elements in these sets of relations from $\mathbb{R}$ and hence plot a Venn diagram containing them all. [10]

$$\begin{aligned}
\mathcal{U} &:= \left\{x^2+2x, x^2-3x+2, \tan(x), x^3, x^{-1}, \cos(x\pi), 3x+1, -|x|, x^2, \frac{x-1}{2}\right\} \\
E &:= \{\text{ relations which have a root at } x=0\} \\
F &:= \{\text{ relations which are onto } \mathbb{R}\} \\
G &:= \{\text{ relations which have value } -1 \text{ at } x=-1\}
\end{aligned}$$

(b) Give a new function which would be in a region of smallest cardinality in your Venn Diagram and check your answer. Find a function which is both onto and one-to-one from $\mathbb{R}$ to the positive real numbers. [2]

Q7. (a) Prove by the contradiction method that two integers sum to an odd number when exactly one of the two numbers is odd. [7]

(b) Give an example of two numbers whose sum is odd but neither are odd. [1]

(c) Determine how many different integers in $\{1, \ldots 13\}$ must be chosen to ensure some pair sums to an odd number. [3]

(d) How many many integers must be chosen to ensure some sum is even? [1]

END OF QUESTION PAPER