Cape Breton University

MATH205

DISCRETE MATHEMATICS

28th October 2009 Time : 1.5 hours

Clearly write your answers to the questions showing all working and checks and indicate what each mathematical calculation is doing. The best THREE answers will be counted.

Q1. (a) Prove, by the contrapositive method, that if $2x^2 + x$ is less than 15 then x is at least -3. [7]

(b) Use real lines to investigate whether these statements are true or false. [4]

$$p(x) \equiv \text{“}|2x - 6| < 7 \text{ ”} , \quad q(x) \equiv \text{“}x \text{ is not positive”}$$

$$\exists x \in \mathbb{Z}; p(x) \wedge q(x) , \quad \forall x \in \mathbb{R}; p(x) \vee q(x)$$

Q2. (a) Use Venn diagrams to show that [5]

$$(A \cup B) \cap (A \triangle D) \subseteq (B \cap D) \cup A)$$

(b) Which sets must be empty for there to be equality in part (a)? [1]

(c) Explain why, for any sets X and Y, it is true that [3]

$$|X \cup Y| \geq |X \cap Y| \text{ and } |X| + |Y| \leq |X \cup Y|$$

(d) How could it be possible that $|X \cup Y| = |X \cap Y|$? [2]

Q3. (a) Prove by induction that $\sum_{i=1}^{n} i^3(3i^2 + 1) = \frac{n^3 (n+1)^3}{2}$. [8]

(b) Use the formula from Handout 2 to get an equation for $\sum_{i=n+1}^{2n} i$ and check if you are right for $n = 0$, 1 and 2. [3]

Q4. Define $p \diamond q$ as $(\sim p) \wedge q$.

(a) Use truth tables to show that $\diamond$ is neither commutative or associative. [3]

(b) Use algebra to determine what the following expressions are in simplest terms. [8]

- $w \diamond (y \diamond w)$
- $\sim (w \diamond y)$
- $(y \rightarrow w) \rightarrow (w \diamond y)$

END OF QUESTION PAPER