

# Math205 Handout 1: Sets and Logic Formulae 2010

- Basic Definitions

Set Theory	Notation	Definition
Complement	$A^c$ (or $\bar{A}$ )	not in
Intersection	$A \cap B$	in both
Union	$A \cup B$	in either <i>or</i> both
Disjoint Union	$A \Delta B$	in either <i>but not</i> both
Universal Set	$\mathcal{U}$	all elements
Empty Set	$\emptyset$	no elements

- Set Relationships

Two sets are equal if they have exactly the same elements in, we write  $X = Y$ .

If element is inside a set we write  $z \in X$ , otherwise we write  $z \notin X$ .

If a set  $X$  is wholly inside of another set  $Y$  we write  $X \subseteq Y$ .

The cardinality of a set  $X$  is written  $|X|$  and is the number of unique elements in  $X$ .

A set is designated  $\{x \mid f(x)\}$  if it is made up of the universal elements  $x$  for which  $f(x)$  is true.

- Set Algebra

The following are the various set relationships we established using Venn Diagrams:

Complementation	$(X^c)^c = X$
Commutativity	$(X \cup Y) = Y \cup X$ $(X \cap Y) = Y \cap X$
Associativity	$(X \cup Y) \cup Z = X \cup (Y \cup Z)$ $(X \cap Y) \cap Z = X \cap (Y \cap Z)$
De Morgan	$(X \cup Y)^c = (X^c \cap Y^c)$ $(X \cap Y)^c = (X^c \cup Y^c)$
Distributive	$(X \cup Y) \cap Z = (X \cap Z) \cup (Y \cap Z)$ $(X \cap Y) \cup Z = (X \cup Z) \cap (Y \cup Z)$
Idempotent	$(X \cup X) = X$ $(X \cap X) = X$
Absorbtion	$(X \cup Y) \cap X = X$ $(X \cap Y) \cup X = X$
Identity	$(X \cup \emptyset) = X$ $(X \cap \mathcal{U}) = X$
Domination	$(X \cap \emptyset) = \emptyset$ $(X \cup \mathcal{U}) = \mathcal{U}$
Inverse	$(X \cup X^c) = \mathcal{U}$ $(X \cap X^c) = \emptyset$

- Inclusion Exclusion

$$|X \cup Y| = |X| + |Y| - |X \cap Y|$$

$$|X \cup Y \cup Z| = |X| + |Y| + |Z| - |X \cap Y| - |X \cap Z| - |Y \cap Z| + |X \cap Y \cap Z|$$

• Basic Definitions

Set Theory	Notation	Definition	Logic	Notation
Complement	$A^c$	not in	Not	$\sim p$
Intersection	$A \cap B$	in both	And	$p \wedge q$
Union	$A \cup B$	in either <i>or</i> both	Or	$p \vee q$
Disjoint Union	$A \Delta B$	in either <i>but not</i> both	Xor	$p \underline{\vee} q$
		If $p$ is true then $q$ is true	Implies	$p \rightarrow q$
		If $p$ and $q$ are the same	If and only if	$p \leftrightarrow q$
Universal Set	$\mathcal{U}$	Always true	Tautology	$T_0$
Empty Set	$\emptyset$	Never true	Absurdity	$(\sim T_0)$

Truth table:

$p$	$q$	$\sim p$	$p \wedge q$	$p \vee q$	$p \underline{\vee} q$	$p \rightarrow q$	$p \leftrightarrow q$	$T_0$
0	0	1	0	0	0	1	1	1
0	1	1	0	1	1	1	0	1
1	0	0	0	1	1	0	0	1
1	1	0	1	1	0	1	1	1

• Logic Relationships

Double Negation	$\sim(\sim p) \equiv p$	
Commutative	$(p \vee q) \equiv (q \vee p)$	$(p \wedge q) \equiv (q \wedge p)$
Associative	$(p \vee q) \vee r \equiv p \vee (q \vee r)$	$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$
De Morgan	$\sim(p \vee q) \equiv (\sim p) \wedge (\sim q)$	$\sim(p \wedge q) \equiv (\sim p) \vee (\sim q)$
Distributive	$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$	$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
Idempotent	$p \vee p \equiv p$	$p \wedge p \equiv p$
Absorbtion	$p \vee (p \wedge q) \equiv p$	$p \wedge (p \vee q) \equiv p$
Identity	$p \vee (\sim T_0) \equiv p$	$p \wedge T_0 \equiv p$
Domination	$p \vee (T_0) \equiv T_0$	$p \wedge (\sim T_0) \equiv (\sim T_0)$
Inverse	$p \vee (\sim p) \equiv T_0$	$p \wedge (\sim p) \equiv (\sim T_0)$
Xor	$p \underline{\vee} q \equiv (\sim(p \wedge q)) \wedge (p \vee q)$ $\equiv ((\sim p) \wedge q) \vee (p \wedge (\sim q))$	
Implication	$p \rightarrow q \equiv (\sim p) \vee q$	
Biconditional	$p \leftrightarrow q \equiv ((\sim p) \vee q) \wedge (p \vee (\sim q))$ $\equiv (p \rightarrow q) \wedge (q \rightarrow p)$ $\equiv (\sim(p \vee q)) \vee (p \wedge q)$	

• Quantifiers

There Exists	$\exists$	
For All	$\forall$	
Negation	$\sim(\forall x (p(x))) \equiv \exists x (\sim p(x))$ $\sim(\exists x (p(x))) \equiv \forall x (\sim p(x))$	
Contrapositive	$\forall x (p(x) \rightarrow q(x)) \equiv \forall x ((\sim q(x)) \rightarrow (\sim p(x)))$	
Equivalences	$\forall x (p(x) \wedge q(x)) \equiv (\forall x p(x)) \wedge (\forall x q(x))$ $\exists x (p(x) \vee q(x)) \equiv (\exists x p(x)) \vee (\exists x q(x))$	
Implications	$\exists x (p(x) \wedge q(x)) \rightarrow (\exists x p(x)) \wedge (\exists x q(x))$ $(\forall x p(x)) \vee (\forall x q(x)) \rightarrow \forall x (p(x) \vee q(x))$	