

27th October 2009

Time : 2-3? hours

Clearly write your answers to the questions showing all reasons, working and checks and indicate what each mathematical calculation is doing. Do not erase anything. Include all rough work and do not commit plagiarism. Feel free to write explanations of what you are thinking at each stage, nothing you can write can lose you marks!

Q1. Let our universal set be the 26 letter English alphabet. Let L be the set of letters that appear in your given names (first and last), let M be the letters which are next to a vowel in the alphabet and let N be the letters from the spelled out words which are three different digits of your registration number.

(a) Identify the members of L , M and N and make a Venn diagram of them. [4]

(b) Create as long a word as possible in a language of your choice from the letters in the set $((L \cup M)^c \cup N)^c$ or explain why it is impossible. [2]

(c) From the eight different disjoint subsets of the Venn diagram identify a subset of either the smallest or the largest cardinality in yours. [1]

Q2. (a) Choose two integers, one odd and the other even. Calculate the difference between the square of the odd one and the sum of the even number and the number one more than it. Is it an integer multiple of 4? [1]

(b) Prove, using the direct method, that the difference between the sum of two consecutive numbers (starting with an even number) and the square of an odd number is always an integer multiple of 4. [6]

Q3. Prove by induction or by manipulation of sums from handout 2 that [7]

$$\sum_{i=1}^n (2i-1)^2 = \frac{n(2n+1)(2n-1)}{3}$$

Q4. Get a truth table for this logic expression and verify that it is equivalent to the most simplified version you can make of this expression. [7]

$$(p \vee q) \rightarrow (r \wedge p)$$

END OF QUESTION PAPER