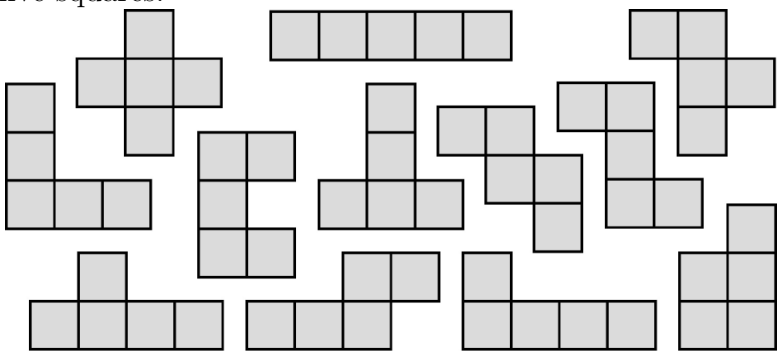
Math2101 Test 1 (September 2011)

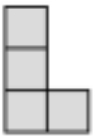
Answer all questions and give complete reasons and checks for your answers. The parts of the questions are weighted as shown and the questions can be answered in any order. Please do not erase any working and hand in your rough work too.

1. Your universal set $\mathcal{U}$ is made up of these pentominoes, which are all possible connected patterns of five squares:



   (a) Determine the contents of these three subsets of $\mathcal{U}$, giving your reasons, and draw a Venn diagram of the three sets:

   A is the set of pentominoes in which there is a square with exactly 3 squares sharing a side with it, B contains the pentominoes which measure longer than 3 squares from one side to another and D is the set of pentominoes which contain the shape L (shown below) or its reflection or rotation. [5]



   (b) Which region R of the Venn diagram is empty? Why is there no hexomino (6 squares connected together) which would be in R? Can there be a shape made up of any other number of squares that would fit in R too? [3]

   (c) What is the cardinality of $A^c \cap (B \cup D)$ for our sets?
   Explain why $|X^c| = |\mathcal{U}| - |X|$ for any set X and hence explain why $|Y^c \cup Z^c| = |Y^c| + |Z^c| - |(Y \cup Z)^c|$ for any sets Y and Z. [4]

2. Simplify this expression using the laws of set algebra and verify your answer using a number of shaded Venn diagrams. [8]

$$((G \cup E^c) \cap (F \cup E)) \cup E$$