Math2101 Test 2 (October 2011)

Answer all questions and give complete reasons and checks for your answers. The parts of the questions are weighted as shown and the questions can be answered in any order. Please do not erase any working and hand in your rough work too.

1. Use the laws of logic algebra to simplify this expression to one with one of each letter and two joining symbols and check your answer with truth tables. [8]

$$(p \to q) \to (r \land p)$$

2. (a) Given these two sets A and B, determine whether or not the following relations are one-to-one or are functions from A to B, giving your reasons. [3]

$$\begin{aligned} A &:= \{\text{three}, \text{six}, \text{nine}, \text{ten}, \text{eleven}\} \\ B &:= \{3, 4, 5, 6\} \end{aligned}$$

- R is the set of pairs (a, b) such that b is the number of letters in the word a.
- S is the set of pairs (a, b) such that the the value of the word is related to its number, if it is in B.
- $T := \{(\text{nine}, 4), (\text{ten}, 6), (\text{six}, 5), (\text{nine}, 3)\}$.

(b) Explain why there cannot be a function from A to B which is both one-to-one and onto, but give a relation that has both of these properties. [1]

3. (a) Identify the regions of the real line for which the following logic statements are true. [4]

$$\begin{aligned} p(x) &:\equiv \text{“ } 2 \le |x - 3| < 4 \text{ ”} \\ q(x) &:\equiv \text{“ } x^2 + x \ge \frac{15}{4} \text{ ”} \\ r(x) &:\equiv \text{“ } |3x - 17| = 2 \text{ ”} \end{aligned}$$

(b) Determine whether or not these quantified statements are true or false by using your number lines from the previous part. In order to show falseness give a value and check that it does not satisfy the full statement. [4]

$$\begin{aligned} \forall x \in \mathbb{Z} \quad &; \quad p(x) \lor q(x) \\ \exists x \in \mathbb{Z} \quad &; \quad q(x) \to r(x) \\ \forall x \in \mathbb{R} \quad &; \quad r(x) \to p(x) \end{aligned}$$