# Math2101 Handout 1: Sets and Logic Formulae 2011

- Basic Definitions

| Set Theory | Notation | Definition |
|---|---|---|
| Complement | $A^c$ (or $\overline{A}$) | not in |
| Intersection | $A \cap B$ | in both |
| Union | $A \cup B$ | in either *or* both |
| Disjoint Union | $A \triangle B$ | in either *but not* both |
| Universal Set | $\mathcal{U}$ | all elements |
| Empty Set | $\varnothing$ | no elements |

- Set Relationships

  Two sets are equal if they have exactly the same elements in, we write $X = Y$.

  If element is inside a set we write $z \in X$, otherwise we write $z \notin X$.

  If a set $X$ is wholly inside of another set $Y$ we write $X \subseteq Y$.

  The cardinality of a set $X$ is written $|X|$ and is the number of unique elements in $X$.

  A set is designated $\{x \mid f(x)\}$ if it is made up of the universal elements $x$ for which $f(x)$ is true.

- Set Algebra

  The following are the various set relationships we established using Venn Diagrams:

| | | | |
|---|---|---|---|
| Complementation | $(X^c)^c$ | $=$ | $X$ |
| Commutativity | $(X \cup Y)$ | $=$ | $Y \cup X$ |
| | $(X \cap Y)$ | $=$ | $Y \cap X$ |
| Associativity | $(X \cup Y) \cup Z$ | $=$ | $X \cup (Y \cup Z)$ |
| | $(X \cap Y) \cap Z$ | $=$ | $X \cap (Y \cap Z)$ |
| De Morgan | $(X \cup Y)^c$ | $=$ | $(X^c \cap Y^c)$ |
| | $(X \cap Y)^c$ | $=$ | $(X^c \cup Y^c)$ |
| Distributive | $(X \cup Y) \cap Z$ | $=$ | $(X \cap Z) \cup (Y \cap Z)$ |
| | $(X \cap Y) \cup Z$ | $=$ | $(X \cup Z) \cap (Y \cup Z)$ |
| Idempotent | $(X \cup X)$ | $=$ | $X$ |
| | $(X \cap X)$ | $=$ | $X$ |
| Absorbtion | $(X \cup Y) \cap X$ | $=$ | $X$ |
| | $(X \cap Y) \cup X$ | $=$ | $X$ |
| Identity | $(X \cup \varnothing)$ | $=$ | $X$ |
| | $(X \cap \mathcal{U})$ | $=$ | $X$ |
| Domination | $(X \cap \varnothing)$ | $=$ | $\varnothing$ |
| | $(X \cup \mathcal{U})$ | $=$ | $\mathcal{U}$ |
| Inverse | $(X \cup X^c)$ | $=$ | $\mathcal{U}$ |
| | $(X \cap X^c)$ | $=$ | $\varnothing$ |

- Inclusion Exclusion

$$|X \cup Y| = |X| + |Y| - |X \cap Y|$$

$$|X \cup Y \cup Z| = |X| + |Y| + |Z| - |X \cap Y| - |X \cap Z| - |Y \cap Z| + |X \cap Y \cap Z|$$

- Basic Definitions

| Set Theory | Notation | Definition | Logic | Notation |
|---|---|---|---|---|
| Complement | $A^c$ | not in | Not | $\sim p$ |
| Intersection | $A \cap B$ | in both | And | $p \wedge q$ |
| Union | $A \cup B$ | in either *or* both | Or | $p \vee q$ |
| Disjoint Union | $A \triangle B$ | in either *but not* both | Xor | $p \veebar q$ |
| | | If $p$ is true then $q$ is true | Implies | $p \to q$ |
| | | If $p$ and $q$ are the same | If and only if | $p \leftrightarrow q$ |
| Universal Set | $\mathcal{U}$ | Always true | Tautology | $T_0$ |
| Empty Set | $\varnothing$ | Never true | Absurdity | $(\sim T_0)$ |

Truth table:

| $p$ | $q$ | $\sim p$ | $p \wedge q$ | $p \vee q$ | $p \veebar q$ | $p \to q$ | $p \leftrightarrow q$ | $T_0$ |
|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 1 | 0 | 0 | 0 | 1 | 1 | 1 |
| 0 | 1 | 1 | 0 | 1 | 1 | 1 | 0 | 1 |
| 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 1 |
| 1 | 1 | 0 | 1 | 1 | 0 | 1 | 1 | 1 |

- Logic Relationships

| | | | | | | |
|---|---|---|---|---|---|---|
| Double Negation | $\sim(\sim p)$ | $\equiv$ | $p$ | | | |
| Commutative | $(p \vee q)$ | $\equiv$ | $(q \vee p)$ | $(p \wedge q)$ | $\equiv$ | $(q \wedge p)$ |
| Associative | $(p \vee q) \vee r$ | $\equiv$ | $p \vee (q \vee r)$ | $(p \wedge q) \wedge r$ | $\equiv$ | $p \wedge (q \wedge r)$ |
| De Morgan | $\sim(p \vee q)$ | $\equiv$ | $(\sim p) \wedge (\sim q)$ | $\sim(p \wedge q)$ | $\equiv$ | $(\sim p) \vee (\sim q)$ |
| Distributive | $p \vee (q \wedge r)$ | $\equiv$ | $(p \vee q) \wedge (p \vee r))$ | $p \wedge (q \vee r)$ | $\equiv$ | $(p \wedge q) \vee (p \wedge r))$ |
| Idempotent | $p \vee p$ | $\equiv$ | $p$ | $p \wedge p$ | $\equiv$ | $p$ |
| Absorbtion | $p \vee (p \wedge q)$ | $\equiv$ | $p$ | $p \wedge (p \vee q)$ | $\equiv$ | $p$ |
| Identity | $p \vee (\sim T_0)$ | $\equiv$ | $p$ | $p \wedge T_0$ | $\equiv$ | $p$ |
| Domination | $p \vee (T_0)$ | $\equiv$ | $T_0$ | $p \wedge (\sim T_0)$ | $\equiv$ | $(\sim T_0)$ |
| Inverse | $p \vee (\sim p)$ | $\equiv$ | $T_0$ | $p \wedge (\sim p)$ | $\equiv$ | $(\sim T_0)$ |
| Xor | $p \veebar q$ | $\equiv$ | $(\sim(p \wedge q)) \wedge (p \vee q)$ | | | |
| | | $\equiv$ | $((\sim p) \wedge q) \vee (p \wedge (\sim q))$ | | | |
| Implication | $p \to q$ | $\equiv$ | $(\sim p) \vee q$ | | | |
| Biconditional | $p \leftrightarrow q$ | $\equiv$ | $((\sim p) \vee q) \wedge (p \vee (\sim q))$ | | | |
| | | $\equiv$ | $(p \to q) \wedge (q \to p)$ | | | |
| | | $\equiv$ | $(\sim(p \vee q)) \vee (p \wedge q)$ | | | |

- Quantifiers

| | | | |
|---|---|---|---|
| There Exists | | $\exists$ | |
| For All | | $\forall$ | |
| Negation | $\sim(\forall x\ (p(x))$ | $\equiv$ | $\exists x\ (\sim p(x))$ |
| | $\sim(\exists x\ (p(x))$ | $\equiv$ | $\forall x\ (\sim p(x))$ |
| Contrapositive | $\forall x\ (p(x) \to q(x))$ | $\equiv$ | $\forall x\ ((\sim q(x)) \to (\sim p(x)))$ |
| Equivalences | $\forall x\ (p(x) \wedge q(x))$ | $\equiv$ | $(\forall x\ p(x)) \wedge (\forall x\ q(x))$ |
| | $\exists x\ (p(x) \vee q(x))$ | $\equiv$ | $(\exists x\ p(x)) \vee (\exists x\ q(x))$ |
| Implications | $\exists x\ (p(x) \wedge q(x))$ | $\to$ | $(\exists x\ p(x)) \wedge (\exists x\ q(x))$ |
| | $(\forall x\ p(x)) \vee (\forall x\ q(x))$ | $\to$ | $\forall x\ (p(x) \vee q(x))$ |