Math2101 Handout 6: Combinatorics

- **Pigeonhole Principle:**

  Given a set of p objects and h groups to place them in, if $h < p$ then at least one group has two objects in it. In general, we can say that <u>one</u> group must have *at least*

  $$\left\lceil \frac{p}{h} \right\rceil$$

  objects in it (those straight brackets round up the nearest integer).

  Note that it can sometimes be useful to round down, in which case the formula is $\left\lfloor \frac{p-1}{h} \right\rfloor + 1$

- **Counting:**

  There are two fundamental rules of counting; multiplicative and additive.

  - *Multiplicative*: if we have two events that are independent of each other and we want to know how many ways both can happen together, we multiply the number of ways each can happen. For instance, to roll a 6-sided die and flip a coin there are twelve outcomes of the form $(d.c)$ where $d \in \{1, 2, 3, 4, 5, 6\}$ and $c \in \{\text{heads}, \text{tails}\}$ and there are $12 = 6 \times 2$ possible such pairs.
  - *Additive*: If we have two events where only one of the two can happen then the total number of events is the sum of the number of ways each event can happen. For instance, if we are adopting one animal from the SPCA and there are 5 dogs and 8 cats at the shelter we have $13 = 5 + 8$ possible choices of a new pet.

  The most general cases of counting involve choosing r objects from a pool of n objects under certain conditions. We have formulae for each of these combinations of conditions;

  - *Repetition*: whether or not we can draw "the same object" again once it has been drawn or not, as if we are replacing the object back in the pool of n objects each time we pick. Note: if we have multiple objects which are indistinguishable and we aren't replacing them, we are still in this case, *assuming* we aren't going to run out of these similar objects.
  - *Order*: whether or not we are picking all r objects at once or in a sequence, such as drawing a hand of cards or picking a soccer team by position.

	Repetition Allowed	Repitition Forbidden
Order Important	n^r a sequence of r rolls of a die n is number of faces of the die	${}_nP_r = \dfrac{n!}{(n-r)!}$ r lotto balls in order n is number of balls
Order Unimportant	$\dbinom{r+n-1}{r} = \dfrac{(r+n-1)!}{r!(n-1)!}$ r arrows, how many in each segment? n is number of segments	$\dbinom{n}{r} = {}_nC_r = \dfrac{n!}{r!(n-r)!}$ hand of r cards n is number of cards