University College of Cape Breton — Math115

Matrix Algebra

February 2004 — Time : 1.5 hours

Answer three whole questions, giving all working and reasoning.

Q1. Find all three eigenvectors of this matrix.

$$\begin{pmatrix} -13 & 10 & 15 \\ -30 & 27 & 45 \\ 10 & -10 & -18 \end{pmatrix}$$

Q2. (a) Given $R := \begin{pmatrix} 2 & x & 1 & 0 \\ -1 & 1 & 0 & 1 \\ 2 & 2 & x-1 & 0 \\ 4x-1 & 1 & 0 & -1 \end{pmatrix}$ evaluate and factorise $\det(R)$.

(b) When $x = 2$ what are all solutions to $R\underline{v} = \underline{0}$?

Q3. (a) Show that if $N := \begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix}$ then $N^2 = 0$.

(b) Find a 2×2 matrix M which isn't a multiple of N which has $M^2 = 0$.

(c) Explain why no $n \times n$ matrix with all positive entries can have $M^2 = 0$.

(d) Using algebra, prove that, if an $n \times n$ matrix M satisfies $M^2 = 0$ then $\det(M) = 0$. Deduce from this that $\operatorname{rank}(M) \leq n - 1$.

(e) Using this information deduce a general form for all possible 2×2 matrices M and give a pattern for an $n \times n$ matrix M with this property which has rank 1.

Q4. Using diagonalisation, find the general solution to the system of equations $p_{k+1} = 42q_k - \frac{61}{2}p_k$ and $q_{k+1} = 31q_k - \frac{45}{2}p_k$ if $p_1 := 2$ and $q_1 := 11$.