Math115 Test 1: 21st January 2004

Show all working and explanations of methods use to achieve full marks.

1. (a) Given the matrix $A := \begin{bmatrix} 1 & -1 & 1 \\ 1 & 1 & 2 \end{bmatrix}$ find the inverse of AA^T and deduce its rank.

   (b) Show that the rank of $A^T A$ is 2 and explain why it cannot have an inverse.

2. (a) Which matrix X satisfies this matrix equation?

$$\left(3X^{-1} + \begin{bmatrix} 1 & -2 \\ 2 & 2 \end{bmatrix}\right)^T = \begin{bmatrix} 4 & -1 \\ -8 & 2 \end{bmatrix}$$

   (b) We define a matrix S as skew-symmetric if $S^T = -S$.

      i. Prove that $(A^T - A)$ is always skew symmetric if A is square.

      ii. Explain why kB will be skew symmetric if B was skew symmetric.

      iii. Find the only $n \times n$ matrix which is both symmetric and skew symmetric.

3. Solve this system of equations fully and give 2 different numerical solutions and check they are indeed solutions.

$$\begin{aligned} 3x_1 + 6x_2 + 2x_3 + 5x_4 + x_5 &= -3 \\ 2x_1 + 4x_2 + 5x_3 + 3x_4 + x_5 &= -8 \\ x_1 + 2x_2 - 3x_3 + 2x_4 &= 5 \end{aligned}$$