Cape Breton University

Math115

MATRIX ALGEBRA

April 2007

Time : 3 hours

Answer FIVE of the SEVEN questions, giving all working and reasoning.

Q1. (a) Find the inverse of $B := \begin{pmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 \end{pmatrix}$ using row operations. [6]

(b) Calculate the determinant of B using Laplace expansions. [3]

(c) Identify the 3×3 submatrix of B corresponding to the largest entry of B^{-1} using the adjoint formula. Calculate its determinant, and check that it gives the expected number. [3]

Q2. (a) Give the underlying matrix of this recurrence: $a_{n+2} = 13a_n + 12a_{n-1}$ [1]

(b) Find the other two eigenvalues given that one is -1. [4]

(c) Use your eigenvalues to form the standard eigenvectors for this type of question. [1]

(d) Give the general expression for $\begin{pmatrix} a_{n+2} \\ a_{n+1} \\ a_n \end{pmatrix}$ in terms of P, D, n, a_2, a_1 and a_0. [1]

(e) You are now given that $a_0 := 0$, $a_1 := 0$ and $a_2 := 7$. Using the adjoint method, or otherwise, find just the three values of P^{-1} necessary (not considering those which will be multiplied by 0) and hence find the general expression for a_n. [5]

Q3. (a) Which value of x will make $C := \begin{pmatrix} 6 & -4 & -4 \\ 2 & 6 & -4 \\ 1 & x & 2 \end{pmatrix}$ singular? [3]

(b) Using this value of x, find the eigenvectors of C. [8]

(c) Explain why all singular matrices will have 0 as an eigenvalue. [1]

Q4. (a) Write these equations in matrix form and take them to an equivalent of Row Echelon Form and hence find all solutions. [8]

$$\begin{aligned} 2p - 4q + 3r + s &= 19 \\ -5p + 3q - 5r - 2s &= -26 \\ 2p - q + 3r + 4s &= 13 \\ -3p - 4q - 2r - 4s &= -1 \end{aligned}$$

(b) Explain why the solutions are in the form of a line in 4 dimensional space. Find the closest distance from this line to the point where $q = 5$ and $p = r = s = 0$. [4]

Q5. A plane T is described by $\begin{pmatrix} w \\ x \\ y \\ z \end{pmatrix} = \alpha \begin{pmatrix} 2 \\ 3 \\ 0 \\ 1 \end{pmatrix} + \beta \begin{pmatrix} 1 \\ -1 \\ -3 \\ 3 \end{pmatrix}$

(a) Show that $f := \begin{pmatrix} 1 \\ 9 \\ 9 \\ -7 \end{pmatrix}$ lies on T but that $g := \begin{pmatrix} 1 \\ -2 \\ 5 \\ 4 \end{pmatrix}$ does not. [4]

(b) Find an equation of the set S of all vectors which are orthogonal to both direction vectors of T and verify that g is in S. [4]

(c) Show that the four vectors that make up S and T are independent. [4]

Q6. (a) Find all quadratics which pass through the points (-1,9) and (1,-4). [4]

(b) Using your result from part (a), explain what happens when you try to find the unique quadratic which passes through (1,-4), (-1,9) and (-3,22). [2]

(c) Find the quadratic which best approximates (-1,9), (1,-4), (-2,3) and (2,5). [6]

Q7. (a) Use Gram-Schmidt on $v_1 := \begin{pmatrix} 3 \\ -2 \\ 6 \end{pmatrix}$, $v_2 := \begin{pmatrix} 0 \\ -1 \\ 2 \end{pmatrix}$ and $v_3 := \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$. [6]

(b) Verify that the matrix Q formed by taking $\frac{e_1}{||e_1||}$, $\frac{e_2}{||e_2||}$ and $\frac{e_3}{||e_3||}$, as its columns has the property $QQ^T = I$ and explain what this means about Q^{-1}. [3]

(c) Explain algebraically why if both Q and R have this property then QR has it. [3]

END OF QUESTION PAPER