Cape Breton University

MATH115

MATRIX ALGEBRA

1st March 2007 Time : 1.5 hours

Clearly write your answers to the questions showing all working and checks and indicate what each mathematical calculation is doing. The best THREE answers will be counted.

Q1. (a) Diagonalise $A := \begin{pmatrix} -30 & -12 \\ 63 & 25 \end{pmatrix}$. [5]

(b) Find the inverses of your D and P. [2]

(c) Multiply the matrices to get $B := PD^{-1}P^{-1}$ and check that $BA = I$. [2]

(d) Explain why $B^{-1} = A$ will be true for any A of any size if it is diagonalisable. [2]

Q2. (a) Find the value of x for which $E := \begin{pmatrix} 7 & 1 & 5 \\ 6 & -1 & x \\ -5 & 3 & 2 \end{pmatrix}$ is singular. [4]

(b) Assuming x is not this value, find the inverse of E using the adjoint method and check your answer by multiplying it by E. [7]

Q3. (a) Determine all solutions to this system of equations: [7]

$$w + 3x - y + 3z = 4 \quad , \quad -w - x + 5y - 3z = 3$$
$$w - 4x + 5y - z = -1 \quad , \quad w + 2x + 2y + 2z = 5$$

(b) Re-use your working from part (a) to find the solution to the corresponding homogeneous equations. [4]

Q4. (a) Rearrage $\alpha(G^T X + H)^{-1} = F$ to make X the subject, one step at a time. [5]

(b) What sizes must F, G and H have for X to possibly be a matrix and why? [2]

(c) If $F := \begin{pmatrix} -1 & 2 \\ 0 & 1 \end{pmatrix}$, $G^T := \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & -1 \end{pmatrix}$, $H := F$ and $\alpha := -2$, find two different possible matrices for X which satisfy our equation. [4]

END OF QUESTION PAPER