

MATRIX ALGEBRA

February 2008

Time : 1.5 hours

Answer THREE of the FOUR questions, giving all working and reasoning.

Q1. (a) Find all solutions for the homogeneous equation of this matrix: [6]

$$A := \begin{pmatrix} 3 & 0 & -3 & -3 & 6 \\ 1 & -2 & -2 & -1 & 5 \\ -3 & -2 & 2 & 3 & -3 \end{pmatrix}$$

(b) What is the rank of A ? Use row operations to find the rank of A^T . Explain briefly why you would expect these two ranks to be equal. [5]

Q2. (a) Diagonalise $J := \begin{pmatrix} 5 & -3 \\ 7 & -5 \end{pmatrix}$ and hence find a general expression for J^n . [7]

(b) Check that the inverse of J is J^{-1} and check that it is also a multiple of J . [2]

(c) Use algebra or guesswork to find another matrix K which is not a multiple of J which also has this exact relation to its inverse. [2]

Q3. (a) Use the laws of matrix algebra to find X if $BXC = C + B$ assuming all other matrices are multipliable and invertible [4]

(b) If X is an $m \times n$ matrix, what size matrices do B and C have to be if we don't insist they have inverses? [3]

(c) Create random 2×1 matrices B and C and find all solutions for X in this case. Does your choice of matrices affect whether or not there are solutions? [4]

Q4. Find all eigenvalues and any two eigenvectors of this matrix. [11]

$$\begin{pmatrix} -43 & 60 & -15 \\ -27 & 38 & -9 \\ 18 & -24 & 8 \end{pmatrix}$$