Cape Breton University

MATH115

MATRIX ALGEBRA

February 2009 Time : 1.5 hours

Please answer any THREE of these questions, please make sure to give all reasoning and working for all questions answered. Start a fresh sheet of paper for each question attempted.

Q1. (a) What are the eigenvalues and eigenvectors of $F := \begin{pmatrix} 5 & -4 \\ 6 & -6 \end{pmatrix}$? [5]

(b) Evaluate F^2 and multiply it by the eigenvectors from part (a) to deduce what the eigenvalues of F^2 are. Explain what you think the eigenvalues of the square of a matrix will be in general, and use algebra to explain why. [4]

(c) Explain why any matrix with determinant 0 will always have eigenvalue 0 too. [2]

Q2. (a) Calculate the adjoint of this matrix: [5]

$$E := \begin{pmatrix} -3 & 4 & 1 \\ 4 & 2 & y \\ -2 & x & -1 \end{pmatrix}$$

(b) Which value of y guarantees that E has an inverse? [4]

(c) Create an always singular matrix including an x and a y and no zeroes. Explain why this cannot be done if the x and y are in different rows and columns. [2]

Q3. (a) Solve this equation for matrix X, using one rule of algebra at a time. Explain what assumptions you make at each step to ensure the equation is solvable: [5]

$$(XA - 3C)^T = A^T B$$

(b) If B is a 2×1 matrix, what sizes would A, C and X have to be? [2]

(c) If all matrices are 2×2 but A has rank 1, choose such A, B and C and find an infinite number of solutions for X. [4]

Q4. (a) Given that the rank of G is 3, find the solution to its homogeneous equation. [8]

$$G := \begin{pmatrix} 11 & 13 & 13 & 23 \\ 10 & 10 & 9 & 6 \\ 1 & 6 & 8 & 13 \\ 3 & 3 & 3 & 15 \\ 7 & 6 & 5 & 7 \end{pmatrix}$$

(b) Find the rank of G^T using new row operations. [3]

END OF QUESTION PAPER