Cape Breton University

MATH115

Matrix Algebra

April 2010

Time : 3 hours

Please answer any FIVE of these questions, please make sure to give all reasoning and working for all questions answered.

Q1. (a) Calculate the determinant of this matrix in two ways, once by a Laplace expansion of a column, then by an expansion of a row; [3]

$$M := \begin{pmatrix} 4 & -3 & 2 \\ y & 4 & x \\ -1 & -1 & -4 \end{pmatrix}$$

(b) If $y = -3$, which value of x would make M singular? Explain why there is no value of x which guarantees that M is always non-singular for every y. [3]

(c) Give the transpose of M and check that the product of it with M is not the same when multiplied on the left or the right for any value of x or y. [3]

(d) Explain algebraically why $3I$, M^2 or the adjoint of M will give the same result when multiplied at the left or the right of M. [3]

Q2. (a) Find the *exact* fit quadratic that passes through (-2,7),(1,0),(3,2) and (0,1). [6]

(b) What is the equation of the best fit straight line for these points? [3]

(c) Find a point which lies on both the quadratic and the line. Would the answers to (a) and (b) would be any different with this point included as a fifth point? [3]

Q3. (a) Use row operations to find the inverse of this matrix: $C := \begin{pmatrix} -2 & 0 & -3 \\ 1 & 4 & 4 \\ -1 & -1 & -2 \end{pmatrix}$ [8]

(b) Use the inverse to find the solution to the equation $CX^T = \begin{pmatrix} 1 & 4 \\ -2 & 1 \\ 1 & 1 \end{pmatrix}$ [3]

(c) What size would Y have to be if Y^TC was an $n \times 3$ matrix? [1]

Q4. (a) Use diagonalisation to find the general solution to this recurrence: [11]

$$b_{i+1} := 3b_i + 28b_{i-1} - 60b_{i-2} \ , \quad b_0 = -8, b_1 = 3, b_2 = -34$$

(b) For which k is it true that $b_{i+1} > b_i$ for all $i \geq k$? Why? [1]

Q5. (a) Check that these vectors are not independent by finding a non-trivial solution to their vanishing equation, using row operations. [6]

$$\left\{ \begin{pmatrix} 2 \\ 0 \\ 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 2 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 0 \\ -2 \end{pmatrix}, \begin{pmatrix} 0 \\ -2 \\ -1 \\ 0 \end{pmatrix} \right\}$$

(b) Use the Gram-Schmidt Process on any 3 of these vectors to find an orthogonal basis. Explain why you would get the all zero vector if you used the fourth vector with the three you already used. [6]

Q6. Let L be the line $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix} + t \begin{pmatrix} -2 \\ 1 \\ -5 \end{pmatrix}$ and P the plane $\begin{pmatrix} x \\ y \\ z \end{pmatrix} \circ \begin{pmatrix} 4 \\ -1 \\ -3 \end{pmatrix} = 7.$

(a) Find the point where L intersects P. [3]

(b) Give four different points on P such that two (A and B) are less than 2 units apart and no three points are in a line. [5]

(c) Find the equation of the lines between A and the third point and B and the fourth point. Where do they meet? Do they have to meet? [4]

Q7. (a) Find and check all three eigenvectors of the matrix $\begin{pmatrix} -1 & 1 & -2 \\ -6 & 4 & -4 \\ -3 & 1 & 0 \end{pmatrix}$. [10]

(b) Give a 2×2 matrix with one eigenvalue appearing twice but only one eigenvector and a similar 3×3 matrix with only two different eigenvectors. [2]

END OF QUESTION PAPER