Cape Breton University

Math115

MATRIX ALGEBRA

February 2010

Time : 1.5 hours

Please answer any THREE of these questions, please make sure to give all reasoning and working for all questions answered. Start a fresh sheet of paper for each question attempted.

Q1. (a) Use a column operation followed by a row operation to find an eigenvalue of this matrix, then factorise the determinant to get the other two eigenvalues: [4]

$$C := \begin{pmatrix} -10 & -12 & 4 \\ -6 & -9 & 2 \\ -42 & -57 & 16 \end{pmatrix}$$

(b) Find an eigenvector by solving the appropriate system of equations. [5]

(c) Without calculating any more matrices, give the eigenvalues of the inverse of C and explain what would happen with the dominant eigenvector for C^{-1} as to how the ratio of the elements of $C^{-k}\underline{v}_0$ would change as k goes to infinity for almost any initial start vector $\underline{v}_0$. [2]

Q2. (a) Given this system of equations and that $a_0 := 1$ and $b_0 := 0$, diagonalise the underlying matrix and hence find a_n and b_n. Check your answers for $n = 0$, $n = 1$ and $n = 2$. [10]

$$a_{i+1} := 146a_i - 168b_i \ , \quad b_{i+1} := 126a_i - 145b_i$$

(b) Explain from your answer to (a) why both sequences will increase at each step apart from once at the beginning. [1]

Q3. (a) Rearrange these equations to the standard form and hence find all solutions: [8]

$$\begin{aligned} 1 + 3\,x &= 3(w + y + z) \\ 4\,x + 6\,y + 5\,z &= 3 + w \\ 3 + 3\,y + z &= x + 2w \\ 5\,w &= 2(2 + x - z) \end{aligned}$$

(b) What is the solution to the equations above when $x = 1$? [1]

(c) Give an example of four equations in four unknowns which have three essentially different homogeneous solutions, give them, and check all three satisfy the homogeneous solution property. [2]

Q4. (a) Find the inverse of $F := \begin{pmatrix} 2 & 2 & 1 \\ 5 & 5 & 3 \\ 1 & 2 & 5 \end{pmatrix}$ using row operations. [6]

(b) Evaluate $\det(F)$ and $\det(F^{-1})$ using Laplace expansions and explain why they are equal. [2]

(c) Explain why a matrix with all of its entries being positive cannot have an inverse with all positive entries. Give an example of a matrix with all non-negative entries whose inverse is also totally non-negative. [3]

END OF QUESTION PAPER