Math115 Test 4: Recurrences and Curve Fitting

March 16th, 2010

Answer all part of the question and give complete reasons and checks for your answers. Attach all rough working and do not erase anything that might earn you marks. The parts of the questions are weighted as shown in square brackets on the right.

1. We will be working with this (x_i, y_i) data throughout this question:

$$(4, 2), (2, -2), (-1, 3), (-3, 4)$$

   (a) Find the quadratic equation which is the least squares best fit to the data. [7]

   (b) Now consider the best fit straight line to the same data. Re-use some of your working from (a) to extract the matrix equation that it will be necessary to solve and hence find the best fit line and how far from each given y_i value it is. [4]

   (c) Explain how and why a best fit straight line may actually get closer to some data points than the quadratic in general, using this question as a reference. Give an example of some points where the quadratic is closer at each x_i value. [2]

2. (a) Use diagonalisation and appropriate shortcuts from the notes given in class to find the general solution to the recurrence which satisfies [6]

$$c_{n+1} := 2c_n + 3c_{n-1} , \quad c_0 := 8 , \quad c_1 := 20$$

   (b) Which n gives the first value of c_n which is over 100 million? [1]