Math115 2011 Test 3

March 2$^{\text{nd}}$, 2011

Answer all questions and give complete reasons and checks for your answers. Please do not erase anything, just put a line through your work and continue; you cannot lose marks for anything you write. The parts of the questions are weighted as shown and can be answered in any order.

1. We will be working with this matrix throughout this question:

$$A := \begin{pmatrix} 5 & -3 & 3 \\ -4 & 9 & -6 \\ -4 & 6 & -3 \end{pmatrix}$$

   (a) Show that $\begin{pmatrix} 6 \\ 3 \\ -1 \end{pmatrix}$ is an eigenvector of A and identify its eigenvalue. [1]

   (b) Find the other eigenvalues of A by using one row and one column operation and a Laplace expansion. [5]

   (c) Find all other eigenvectors of A. [6]

2. In this question this will be the matrix under investigation.

$$B := \begin{pmatrix} -2 & x & -1 \\ 5 & -5 & 2 \\ -1 & 4 & y \end{pmatrix}$$

   (a) For which value of x is B guaranteed to be non-singular? [3]

   (b) Assuming x is this value, calculate the adjoint of B and explain why the determinant of B^{-1} will be $\frac{-1}{3}$ in this case. [5]